

Name: _____

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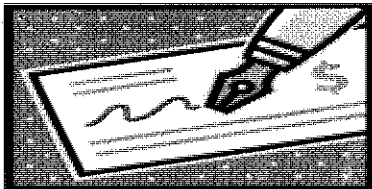
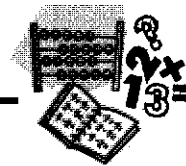
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Math 9 K&E

Whole Numbers (A)

Numeric and Word Forms



Recognizing and writing word forms of numbers is important in some workplace situations and when writing cheques.

Numbers can be represented in two ways.

1. In **numeric** form e.g., 10, 25, 179
2. In **word** form e.g., ten, twenty-five, one hundred seventy-nine

Numbers are written in words by identifying where each number is located in the place value chart, starting from **left to right**.

REMEMBER:

Place a **hyphen (-)** between the **tens** and **ones** place values.

E.g., twenty-four, four thousand seventy-nine

Examples

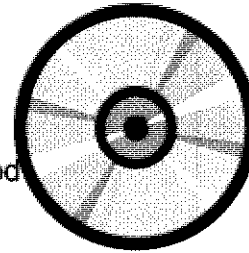
A) Look at the following numeric and word forms.

- 1400 in writing is one thousand four hundred.
- 23 756 in writing is twenty-three thousand seven hundred fifty-six.
- 45 921 in writing is forty-five thousand nine hundred twenty-one.
- 762 367 in writing is seven hundred sixty-two thousand three hundred sixty-seven.
- 3 436 188 in writing is three million four hundred thirty-six thousand one hundred eighty-eight.

These numbers are shown in the place value chart below.

One Millions	Hundred Thousands	Ten Thousands	One Thousands	Hundreds	Tens	Ones
			1	4	0	0
		2	3	7	5	6
		4	5	9	2	1
	7	6	2	3	6	7
3	4	3	6	1	8	8

- B) The music and recording industry sell millions of dollars worth of music CDs each year.



Just imagine the sales they will have over a ten-year period and the large numbers they work with!

The numbers below have been placed in the place value chart.

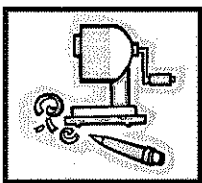
35 000 460 950 3 000 000 72 500 000 250 634 750

8 325 400 000 15 000 005 000 675 000 000 000

billions			millions			thousands			units		
<i>hundreds</i>	<i>tens</i>	<i>ones</i>	<i>hundreds</i>	<i>tens</i>	<i>ones</i>	<i>hundreds</i>	<i>tens</i>	<i>ones</i>	<i>hundreds</i>	<i>tens</i>	<i>ones</i>
							3	5	0	0	0
						4	6	0	9	5	0
					3	0	0	0	0	0	0
				7	2	5	0	0	0	0	0
			2	5	0	6	3	4	7	5	0
		8	3	2	5	4	0	0	0	0	0
	1	5	0	0	0	0	0	5	0	0	0
6	7	5	0	0	0	0	0	0	0	0	0

These numbers in **word form** are written in order from smallest to largest as follows:

35 000	thirty-five thousand
460 950	four hundred sixty thousand nine hundred fifty
3 000 000	three million
72 500 000	seventy-two million five hundred thousand
250 634 750	two hundred fifty million six hundred thirty-four thousand seven hundred fifty
8 325 400 000	eight billion three hundred twenty-five million four hundred thousand
15 000 005 000	fifteen billion five thousand
675 000 000 000	six hundred seventy-five billion.



Practice: Numeric and Word Forms

1. Fill in the missing information for each of the charts.

Number	Number in Words
	forty-eight
	fifty
53	
	fifty-six
61	
51	
	sixty-two
67	
70	
	eighty-seven
94	

Number	Number in Words
	two hundred twenty-seven
	eight hundred forty-six
	six hundred nine
	four hundred
	seven hundred fifty-two
	three hundred four
	nine hundred thirty-one

Number	Number in Words
425	
172	
551	
308	
999	
612	
801	
535	
240	

2. Practise writing numbers in numeric and word forms by filling in the missing values in these cheques.

\$

 One thousand two hundred sixty

\$

 DOLLARS

\$

 Twenty-two thousand four hundred fifty-five

\$

 DOLLARS

PAY TO THE ORDER OF

Four hundred thirty-two thousand

19

\$

DOLLARS

FOR

PAY TO THE ORDER OF

Six hundred fifty-three thousand two hundred ninety-five

19

\$

DOLLARS

FOR

PAY TO THE ORDER OF

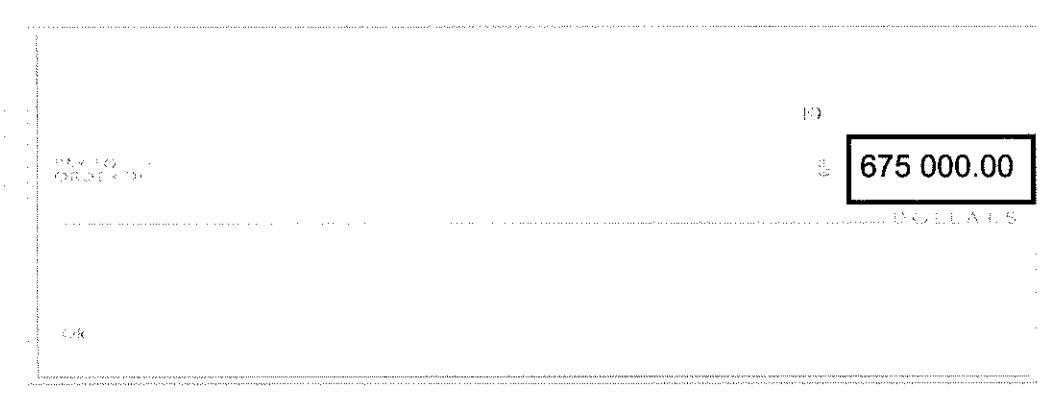
2350.00

19

\$

DOLLARS

FOR



3. The table below shows the population of each of Canada's provinces and territories. Write each population total out in word form.

Province/Territory	Population: Numeric Form	Population: Word Form
British Columbia	3 907 738	
Alberta	2 974 807	
Saskatchewan	978 933	
Manitoba	1 119 583	
Ontario	11 410 046	
Quebec	7 237 479	
New Brunswick	729 498	
Nova Scotia	908 007	
Prince Edward Island	135 294	
Newfoundland	512 930	
Yukon Territory	28 674	
Northwest Territories	37 360	
Nunavut	26 745	

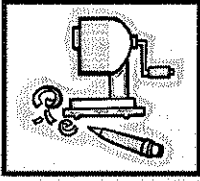
4. Pick three of the numbers from each chart and try to say them out loud. Then, write out all of the numbers in **word form** on a separate piece of paper.

Table A

One Millions	Hundred Thousands	Ten Thousands	One Thousands	Hundreds	Tens	Ones
			5	2	5	3
		6	1	3	2	5
		9	4	7	3	8
	8	7	2	1	9	5
	1	2	5	6	0	4
	9	3	8	8	4	1
2	3	1	7	5	3	2

Table B

billions			millions			thousands			units		
<i>hundreds</i>	<i>tens</i>	<i>ones</i>	<i>hundreds</i>	<i>tens</i>	<i>ones</i>	<i>hundreds</i>	<i>tens</i>	<i>ones</i>	<i>hundreds</i>	<i>tens</i>	<i>ones</i>
					1	5	4	0	0	0	0
				9	3	2	0	0	0	0	0
			2	8	6	5	4	0	0	0	0
		6	0	0	0	0	0	0	0	0	0
	7	0	5	0	0	0	0	0	0	0	0
4	2	3	1	3	1	6	0	0	0	0	0



Practice: Solving Word Problems with Numeric and Word Forms

Use a place value chart to help you with these large numbers.

1. A village is defined as a location that has between 1 and 999 people.
A 2001 census lists the populations of these three Alberta villages as:

Alliance – 169 people
Forestburg – 930 people
Consort – 696 people.

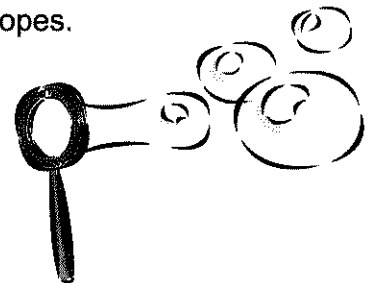
Write each of these population figures out in words.
Round each to the nearest hundred and add them together.

2. Karen got a job tree planting in northern Alberta. Her crew was responsible for planting trees in an area of 5 km^2 . Karen's group was required to plant eight thousand seven hundred twelve spruce trees, five thousand four hundred thirty evergreen trees, and two thousand five hundred one poplars.

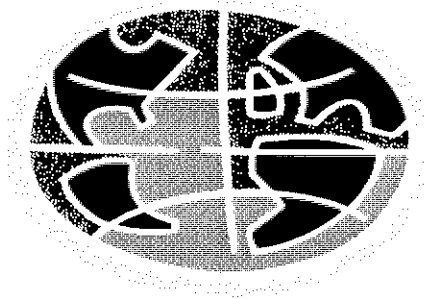
- a) Write out all the numbers in numeric form.
- b) How many more spruce trees were being planted than poplars?
- c) How many trees were being planted in total?

3. Jasmine's manufacturing company received two written invoices from a huge toy store. The company placed an order for 28 565 kites and 60 029 bubble-makers. A few weeks later they placed an order for twenty-eight thousand one hundred seventeen hoops and sixty thousand twenty-eight skipping ropes.

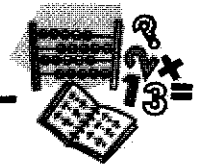
- a) Write out all the numbers in numeric form.
- b) Which order was greater: the bubble-makers and the kites or the skipping ropes and the hoops?
- c) By how much?



4. In July 2000, the population of the United States was approximately two hundred seventy-five million six hundred seventy-three. The population of China was one billion two hundred sixty-one million eight hundred thirty-two thousand four hundred eighty-two and the population of Canada was thirty-one million two hundred eighty-one thousand ninety-two.
- a) Write out all the numbers in numeric form.
- b) Estimate how much greater the population of China was than the populations of both Canada and the United States. Write your answer in numeric and word form.



Place Value of Whole Numbers



Understanding place value is important for everyone who handles money: bank tellers, grocery store cashiers and even you!

Example

Jason received this paycheque from his part-time job. Take a closer look to see how much he earned.

ACME Ltd.	#042
Pay to the order of _____ Jason _____	
The sum of: <u>One thousand six hundred fifty-two dollars</u>	\$1652.00
Signed <u>Employer</u>	

Jason's paycheque shows that he made \$1652.00!
This amount can be shown as follows:

Type of Bill	Number of Bills and Coins	Number of Bills and Coins
\$1000 bills	1	
\$100 bills	6	
\$10 bills	5	
\$1 coins	2	

This break down of money uses PLACE VALUE.

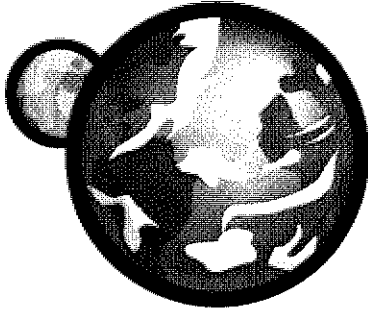
Jason has:

- 1 bill in the **thousands** position
- 6 bills in the **hundreds** position
- 5 bills in the **tens** position
- 2 coins in the **ones** position.

Place Value and Large Numbers

Sometimes we need to recognize the value of large numbers to understand the world around us.

Examples

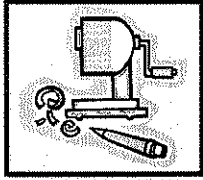


The distance between the Earth and the moon is about 385 000 000 m.

Zeros are used as place holders.

The circumference of the earth is approximately 39 930 000 m.

The circumference of the moon is approximately 6 000 000 m.



Practice: Identifying Place Value

1. Look at the following paycheque:

ACME Ltd.	#043
Pay to the order of _____ Jason _____	
The sum of: <u>One thousand five hundred twenty-six dollars</u>	\$1526.00
Signed <u>Employer</u> _____	

How much did Jason earn? Say the number out loud.

How many of each bill and how many coins did Jason earn?

Type of Bill/Coin	Number of Bills and Coins
\$1000 bills	
\$100 bills	
\$10 bills	
\$1 coins	

2. Give this one a try!

ACME Ltd.	#044
Pay to the order of _____ Jason _____	
The sum of: <u>Two thousand four hundred thirty-five dollars</u>	\$2435.00
Signed <u>Employer</u>	

How much did Jason earn? Say the number out loud.

How many of each bill and how many coins did Jason earn?

Type of Bill/Coin	Number of Bills and Coins
\$1000 bills	
\$100 bills	
\$10 bills	
\$1 coins	

3. Fill in the amounts in their correct place values. The first one is done for you.

Cheque Amount	Place Value			
	thousands	hundreds	tens	ones
\$1652	1	6	5	2
\$4526				
\$2742				
\$6135				
\$1978				
\$3024				
\$2007				
\$7800				

4. Say each number out loud. Then, identify the place value of the underlined numbers.

Number	Place Value of Underlined Number
34 <u>5</u> 6	
6 <u>3</u> 15	
75 <u>2</u> 5	
<u>2</u> 100	
8 <u>9</u> 75	

5. Place the circumferences of the earth and moon in the chart below. Say each number out loud to yourself.

	Hundred Millions	Ten Millions	One Millions	Hundred Thousands	Ten Thousands	One Thousands	Hundreds	Tens	Ones
385 000 000	3	8	5	0	0	0	0	0	0
39 930 000									
6 000 000									

6. Locate several sources of information involving large numbers, such as distances, world populations and sizes of rivers, lakes and buildings. Challenge your classmates to place them on a place value chart correctly.

Sources of information include atlases, encyclopedias, almanacs, newspapers, libraries and Internet sites.

7. The left column of the table below shows the amount of money collected from charity drives for the last six years. Complete the place value chart for the total of each year.

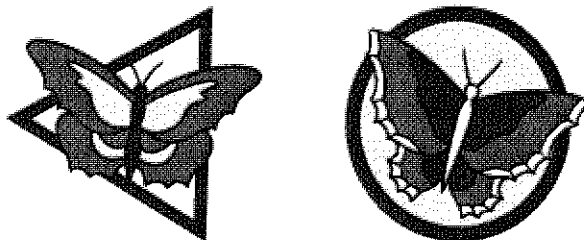
Money Collected (\$)	Hundred Millions	Ten Millions	One Millions	Hundred Thousands	Ten Thousands	One Thousands	Hundreds	Tens	Ones
775 525									
32 100 000									
102 500 450									
7 500 000									
143 600 250									
74 832 195									

8. Place the following numbers in the place value chart below.

a) 4 000 000 000 b) 6 750 624 128 c) 1 032 507 125 d) 3 025 456 980

Place Value										
	One Billions	Hundred Millions	Ten Millions	One Millions	Hundred Thousands	Ten Thousands	One Thousands	Hundreds	Tens	Ones
a)										
b)										
c)										
d)										

Comparing and Ordering Whole Numbers



Sometimes it is not easy to tell one butterfly from another. Sometimes it is difficult to tell numbers apart, too.

To compare numbers with the same number of digits:

- Determine which has the larger **first number** (the digit furthest left).

For example, 420 is larger than 240 because 4 is larger than 2.

To compare numbers that have a different number of digits:

- Look at the **place values** of each number. The number that extends the furthest to the left is the larger number.

For example, 240 is larger than 85 because 2 is further to the left than 8 when placed on a place value chart. The number 2 in 240 has a higher place value.

Example



Comparing and ordering the populations of countries around the world involves working with large numbers!

Ravneet and Azim noted the following numbers from their textbook:

13 395 000 15 058 600 20 100 000

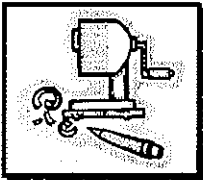
A place value chart, like the one below, can help order and compare the numbers recorded by Ravneet and Azim.

Place Value									
One Billions	Hundred Millions	Ten Millions	One Millions	Hundred Thousands	Ten Thousands	One Thousands	Hundreds	Tens	Ones
		1	3	3	9	5	0	0	0
		1	5	0	5	8	6	0	0
		2	0	1	0	0	0	0	0

2 has a value greater than 1, so 20 100 000 has the greatest value.

5 has a value greater than 3, so 15 058 600 has the second greatest value.

$$20\ 100\ 000 > 15\ 058\ 600 > 13\ 395\ 000$$



Practice: Comparing and Ordering Whole Numbers

- Bill is keeping a log of the books that he reads and the number of pages in each book. Here is his list for the month of May:

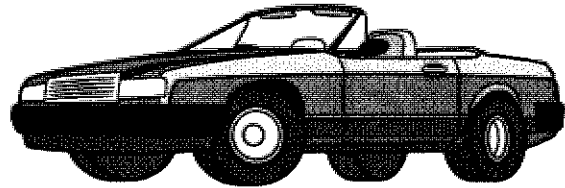
The Macabre Incident — 1205 pages
Screaming Danger — 786 pages
Fighting Times — 876 pages
Terror Below — 1148 pages
Mission Achieved — 250 pages
Awesome Antics — 184 pages.



Put these books in order from least to greatest according to the number of pages.

2. Melba is pricing used cars. She is interested in the following models:

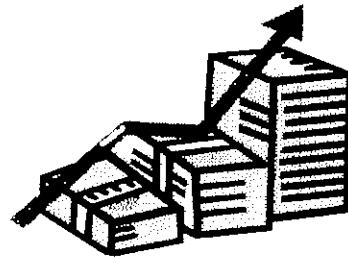
1994 Honda	\$2500.00
1999 Mazda	\$7850.00
1998 Datsun	\$5025.00
2000 Saturn	\$8650.00
1996 Toyota	\$3600.00



Place the cars in order from highest price to lowest price.

3. Jody's social studies class is studying economics. Students were asked to form groups and invest the same amount of pretend money in a variety of stocks. Jody's group selected 6 different stocks and, by the end of the activity, had earned the following amounts of money:

Investment 1	\$ 7635.00
Investment 2	\$ 17 672.00
Investment 3	\$ 92 450.00
Investment 4	\$ 6892.00
Investment 5	\$ 89 032.00
Investment 6	\$ 28 782.00



Which investment was the best choice during this period of time?
Which was the poorest choice?

List the investments in order from the one that made the least amount of money to the one that made the greatest amount of money.

4. Find a partner to work with. Each of you needs a copy of the charts below.



The Challenge: Race to the finish!

The first person to fill in both charts correctly wins the race. Compare your answers with your partner. Discuss and correct any errors.

Complete the following chart by filling in the number that belongs in each place value.

	Place			Value					
	Hundred Millions	Ten Millions	One Millions	Hundred Thousands	Ten Thousands	One Thousands	Hundreds	Tens	Ones
2500						2	5	0	0
10 300									
1 650 975									
840 000									
3 000 000									
210 000 000									
52 000									

Place the values in order from LEAST to GREATEST in the chart below. Be sure to include the example.

Least

Greatest

--	--	--	--	--	--	--

5. Place the numbers in order from LEAST to GREATEST.

a. 2340 3564 2198 2975 3529

b. 23 642 23 562 21 728 20 953 32 529

c. 312 320 431 565 426 190 322 112 398 945

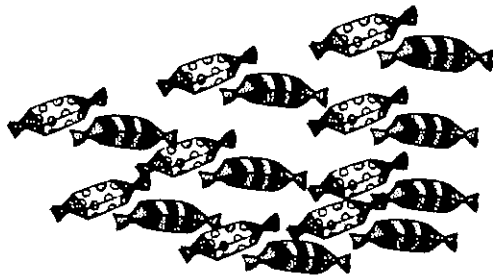
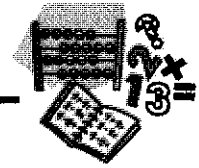
6. Place the numbers in DECREASING ORDER.

a. 1 435 612 3 100 395 3 099 875 1 299 580 2 355 315

b. 63 745 324 63 345 743 73 126 843 61 737 843 72 513 853

c. 3 435 612 5 100 395 5 099 875 3 299 580 4 355 315

Odd and Even Numbers



If the candies to the left are divided between two people, will each person get the same number of candies?

The answer depends on whether the number of candies is odd or even.



When we can put numbers into groups of two without having any left over, they are called **even numbers**.

When we cannot put numbers into groups of two, they are called **odd numbers**.

Strategies for Figuring Out if a Number is Odd or Even

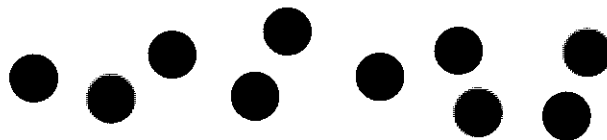
Use Manipulatives



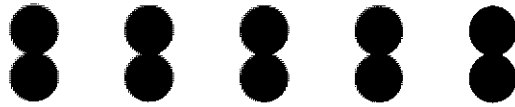
Use checkers or other manipulatives. Take the number you are trying to figure out and put them into groups of 2. If there are none left over, the number is even. If there is one left over, the number is odd.

Example

Find **10 checkers** or other manipulatives and place them flat on the desk in front of you. Do not stack them.



Put the 10 checkers into groups of 2 so that they look like this:



You have 5 groups with 2 checkers each. This means that 10 is an even number.

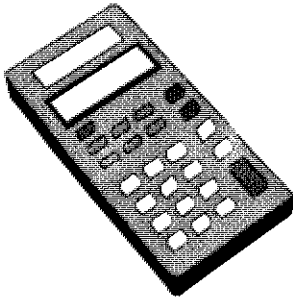
Now remove any 3 checkers so that you have 7. Place the remaining checkers in groups of 2 again.



There is one extra checker left over. This means that 7 is an odd number.

Use Your Calculator

Enter the number. Press the \div key and then the 2 key.



- If the number displayed is not a decimal number, then the number is **EVEN**.

Example

5

- If the number displayed is a decimal number, then the number is **ODD**.

Example

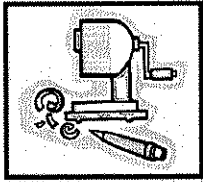
2.53

Look at the Last Digit

If the last digit of a number can be divided by two, the number is **even**.

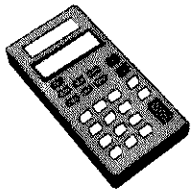
Example

6 can be divided by 2, so 2416 is even
BUT 7 cannot be divided by 2, so 2417 is odd.



Practice: Odd and Even Numbers

1. Use your calculator to determine if this number is odd or even: **103 726**



1	0	3	7	2	6	÷	2	=
---	---	---	---	---	---	---	---	---

2. Find a partner. Place 29 paper clips or other objects on a flat surface. Take turns removing paper clips from the pile. You may remove 1, 2, 3 or 4 paper clips at a time. The player who finishes the game with an even number of paper clips wins the game.



Play the game a few times and then devise a strategy so that you will always win, regardless of whether you go first or second.

Create other similar games to challenge your classmates.

3. Individually or with a partner, complete a variety of activities to prove whether numbers are even or odd.

Multiples



Multiples are numbers that can be **divided evenly** by other numbers. Multiples are also the product when two or more factors are multiplied.

Examples

- A) 15 is a multiple of 3 and 5, because $3 \times 5 = 15$
B) 24 is a multiple of 8 and 3, because $8 \times 3 = 24$

Strategies for Determining Multiples

Multiplication using mind math or pencil and paper

Example

Identify the first five multiples of 7. Think of your multiplication facts.



$$\begin{aligned}7 \times 1 &= 7 \\7 \times 2 &= 14 \\7 \times 3 &= 21 \\7 \times 4 &= 28 \\7 \times 5 &= 35\end{aligned}$$

The first five multiples of 7 are:

7, 14, 21, 28, 35

You can determine multiples of numbers by skip counting. Think of multiplication facts to help you. Count up to find the next three multiples of these numbers:

4 4, 8, 12, 16, 20, 24, 28

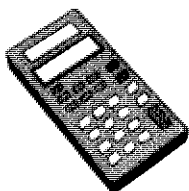
9 9, 18, 27, 36, 45, 54

12 12, 24, 36, 48, 60, 72, 84

Addition using a calculator

Example

Identify the first five multiples of 3.



1. Turn on your calculator.
2. Type in the number **3**.
3. Then hit the **+** key.
4. Now hit the **3** key (the multiple you are counting up by).
5. Hit the **=** key and write down the number. Hit the **=** two more times, and write down each number displayed in order.

The first six multiples of 3 are:

3 6 9 12 15

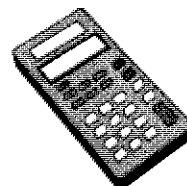
Multiplication using a calculator

Your calculator can also find multiples of numbers using multiplication.

Example

Find the first five multiples of 8.

1. Turn on your calculator.
2. Type in the number **8**.
3. Then hit the **×** key.
4. Now hit the **1** key.
5. Hit the **=** key and write down the number.
 $8 \times 1 = 8$
6. Repeat steps 1 to 5, but increase the value of the key in step 4 by 1 each time



$$8 \times 2 = 16$$

$$8 \times 3 = 24$$

$$8 \times 4 = 32$$

$$8 \times 5 = 40$$

The first five multiples of 8 are:

8 16 24 32 40

Sets of Multiples

The multiples of a number are represented as sets, lists or groups of numbers.

Example

The first six multiples of 8 are 8, 16, 24, 32, 40, 48.

The set of the first six multiples of 8 is:

8: {8, 16, 24, 32, 40, 48}

colon

Multiples of 8 in increasing order from left to right separated by commas.

The { } signs show that numbers inside belong to a set of numbers.

If a specific number of digits in a set is not asked for, write the set of multiples like this:

The set of multiples of 6 is:

6: {6, 12, 18 ...}

Write the first 3 multiples in order.

Write **three periods** to indicate that the pattern continues.

Lowest Common Multiples



Hamburger patties and buns are often sold in different quantities.

To purchase exactly the same number of patties and buns, we could calculate the lowest common multiple of each package of buns and hamburger patties.



The **Lowest Common Multiple (LCM)** of a pair of numbers is the smallest multiple that is common to both the numbers.

Follow these steps to find the lowest common multiple of two numbers.

1. List multiples of both numbers in order from least to greatest. (You do not need to list all the possible multiples for both numbers. List a few at a time for each number until the same multiple appears in both lists.)
2. Determine the lowest multiple that is the same for both numbers.

Example

If hamburger patties were sold in packages of 6, and buns in bags of 8, how many would we need to buy to have the same number of patties and buns?

Find the **lowest common multiple** of 6 and 8.

Multiples of 6 include: 6: {6, 12, 18, 24, 30...}

Multiples of 8 include: 8: {8, 16, 24, 32 ...}



The lowest common multiple of 6 and 8 is **24**

We would need to purchase 24 patties and 24 buns to have an equal number of both.

Remember there are 6 patties in a package.

$$24 \div 6 = 4$$

$$24 \div 8 = 3$$

There are 8 buns in a bag.

We would need 4 packages of hamburger patties and 3 packages of buns.

Using Multiples to Solve Problems

Examples

- A) The number 22 bus comes to James' bus stop every 12 minutes. The number 27 arrives every 8 minutes. In how many minutes will both buses arrive at the stop together?

Find the multiples of 12 and 8.

Multiples of 12 are 12, 24, 36, etc.

Multiples of 8 are 8, 16, 24, etc.

Conclusion: Both buses will arrive at James' stop together every 24 minutes.

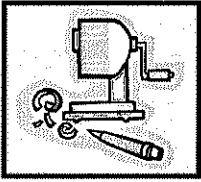
- B) Bonnie rotates the tires of her truck every 8000 km and changes the oil every 5000 km. She will rotate the tires and change the oil at the same time after how many kilometres?

Find the multiples of 8000 and 5000. (Hint: Use 8 and 5)

Multiples of 8 are 8, 16, 24, 32, 40, etc.

Multiples of 5 are 5, 10, 15, 20, 25, 30, 35, 40, etc.

Conclusion: Bonnie will rotate her tires and change her oil together every 40 000 km.



Practice: Finding Multiples and Sets of Multiples

1. Use your calculator or mind math to find the missing multiples. Then, write each group of multiples as a set.

4

12

6

100

2. Fill in the missing numbers and then answer each question.

1 2 3 4 6 8

This code was cracked by counting by multiples of _____.

2 4 6 10 12 16

This code was cracked by counting by multiples of _____.

5 10 20 25 35

This code was cracked by counting by multiples of _____.

10	20	30		50	60		80
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This code was cracked by counting by multiples of _____.

20	40	60	80		120	140	
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This code was cracked by counting by multiples of _____.

3. The first three multiples of 6 are listed below. Fill in the next three and then finish writing the multiples as a set by filling in the blanks.

$$6 \times 1 = 6$$

$$6 \times 2 = 12$$

$$6 \times 3 = 18$$

$$6 \times 4 =$$

$$6 \times 5 =$$

$$6 \times 6 =$$

$$6 : \{ 6, 12, 18, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$$

4. The first three multiples of 12 are listed below. Fill in the next three and then finish writing the multiples as a set by filling in the blanks.

$$12 \times 1 = 12$$

$$12 \times 2 = 24$$

$$12 \times 3 = 36$$

$$12 \times 4 =$$

$$12 \times 5 =$$

$$12 \times 6 =$$

$$12 : \{ 12, 24, 36, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$$

5. Is 72 a multiple of 9? Show your response.

6. Identify the 2 sets of numbers (all less than 10) that have a lowest common multiple of 18.

7. Identify the 2 numbers (less than 10) that have a lowest common multiple of:

a) 12 E.g. 6 and 2 _____

_____ 4 and 3 _____

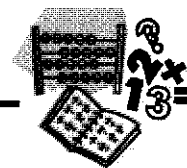
b) 20 _____

c) 15 _____

d) 32 _____

8. The school band needs refreshments and snacks for all 58 band members who will be rehearsing Saturday afternoon. Pop comes in cases of 12 and muffins come in packages of 6. Find the lowest common multiple of 12 and 6 that will give all 58 students a pop and muffin on Saturday.

Factors



Numbers that are multiplied together to give you another number are **factors** of that number.

Example

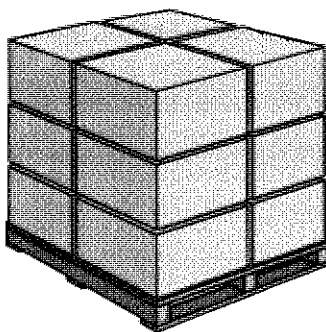
$$6 \times 5 = 30$$

6 and 5 are factors of 30.

Remember that the product of two numbers is called a multiple.

$$6 \text{ (factor)} \times 5 \text{ (factor)} = 30 \text{ (multiple)}$$

30 is a multiple of both 5 and 6.



Factors are useful when determining how many different ways these boxes can be stacked and arranged to be quickly counted in a warehouse.

Numbers can be broken down to their factors. For example:

Represent 6 by showing the factors of 6:

$$\begin{array}{l} 6 \times 1 \\ 3 \times 2 \end{array} \quad \Rightarrow \quad 1, 2, 3 \text{ and } 6 \text{ are all factors of } 6.$$

Represent 24 by showing the factors of 24:

$$\begin{array}{l} 24 \times 1 \\ 12 \times 2 \\ 6 \times 4 \\ 8 \times 3 \end{array} \quad \Rightarrow \quad 1, 2, 3, 4, 6, 8 \text{ and } 12 \text{ are all factors of } 24.$$

Finding the Factors of a Number

You can use the steps below to find the factors of any whole number.

Step 1 1 and the number are always factors.

Step 2 Divide the number by 2 and see if you get a number without a decimal.

- If you get a whole number, then 2 and the whole number you get are both factors of the number.
- If you get a decimal number, then 2 is not a factor.

Step 3 Keep dividing by consecutive numbers (3, 4, 5, 6, etc.) until you have found all the factors.



You will know when all the factors have been found when you divide by a number that you already have as a factor, or when you get the same number as the number you are dividing by. Each factor should only be listed once.

Examples

A) Identify the factors of 64



$$64 \div 1 = 64$$

$$64 \div 2 = 32$$

$$64 \div 3 = \text{decimal number}$$

$$64 \div 4 = 16$$

$$64 \div 5 = \text{decimal number}$$

$$64 \div 6 = \text{decimal number}$$

$$64 \div 7 = \text{decimal number}$$

$$64 \div 8 = 8$$

1 and 64 are pairs of factors of 64

2 and 32 are pairs of factors of 64

3 is not a factor of 64

4 and 16 are pairs of factors of 64

5 is not a factor of 64

6 is not a factor of 64

7 is not a factor of 64

8 is a factor of 64

The set of factors of a number are listed in order from LEAST to GREATEST and separated by commas.

The set of factors of 64 is: 64: {1, 2, 4, 8, 16, 32, 64}

Notice that $8 \times 8 = 64$, but 8 is only listed once in the list of factors.

B) Is 5 a factor of 125?

Looking at Hints for Dividing, we see the following.

Rule: 5 is a factor if the number ends in a 0 or 5.

Conclusion: 5 is a factor of 125.

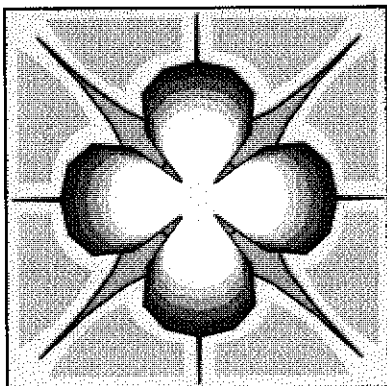
To find another factor, we can divide 125 by 5.

$$125 \div 5 = 25$$

Conclusion: 5 and 25 are both factors of 125

Greatest Common Factors

Common factors are used in many ways. For example, we use common factors when deciding on the largest size of tile to cover a floor without having any left over.



Example

What is the largest size of tile that will cover a 6 feet \times 4 feet floor?

Factors of 6 are 1, 2, 3 and 6.

Factors of 4 are 1, 2 and 4.

The largest size of tile to cover the floor would be 2 feet by 2 feet.



The **Greatest Common Factor (GCF)** of a pair of numbers is the largest factor found in the factors sets of both the numbers.

Follow these steps to find the greatest common factor of two numbers.

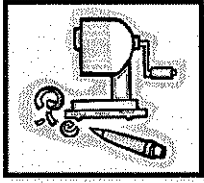
1. List **ALL** the factors for both numbers in order from least to greatest.
2. Determine the **largest** factor that is common to both sets.

Example

Find the **greatest common factor** of 20 and 45.

The factors of 20 are: 1, 2, 4, 5, 10, 20.
The factors of 45 are: 1, 3, 5, 9, 15, 45.

→ The greatest common factor of 20 and 45 is 5.



Practice: Finding Factors

1. A factor is missing in each of the questions below.
Write in the missing factor.

a. $2 \times \square = 24$

d. $7 \times \square = 56$

b. $\square \times 12 = 48$

e. $\square \times 7 = 77$

c. $9 \times \square = 108$

f. $\square \times 4 = 64$

2. Use a calculator or another method to list the set of factors for the following numbers.

a. 20 E.g., 20: {1, 2, 4, 5, 10, 20} _____

b. 45 _____

c. 36 _____

d. 80 _____

- e. 24 _____
- f. 51 _____
- g. 90 _____
- h. 33 _____
- i. 48 _____
- j. 56 _____

3. Name 3 sets of numbers (all less than 50) that have the same greatest common factor.
4. Name the factors of each number. Find the greatest common factor of each pair of numbers

	<u>Factors</u>	<u>Number</u>	<u>Greatest Common Factor</u>
a)	E.g. 1, 2, 3, 4, 6, 9, 12, 18, 36 1, 2, 4, 7, 14, 28	36 28	4
b)		21 49	
c)		16 40	

5. The art class is creating a wall mural. They want to cut a large, rectangular piece of wood into equal pieces. Some art students will then take a piece to paint for the mural. The wood is 14 feet by 6 feet. What is the largest size of piece the students can cut to make all pieces equal?
6. The period 2 CALM classes are meeting in the common room to hear a career speaker. The speaker would like tables with equal groups of students. If class B2 has 32 students and class D2 has 28 students, what number of students should sit at each table for the least amount of tables?

