Name:_____

Start date:_____

Hand in date:_____

Mark:_____

Math 9 K&E

Patterns and Relations

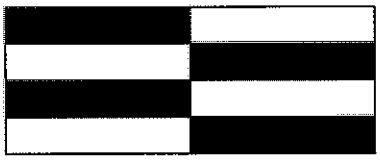
Patterns

The above pictures represent the four seasons. Seasons follow a pattern.

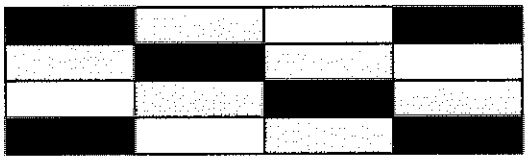
Patterns exist in nature and in everyday life. **Patterns** occur over and over again; patterns show relationships.

Some everyday patterns are days of the week and months in a year. The growth of plants also forms a pattern. For example, seed, germination, growth, flower and seed. Cause and effect relationships also form patterns. For example, preparing for tests by doing homework and studying leads to an improved mark.

Here is another example of a pattern: The dark and light shadings alternate on each row.



In this pattern, the shadings alternate,
dark, medium and light. All diagonal squares (↘) have the same shading.



Think About ...

What patterns do you see around you ...
... in your home life?
... in your school?
... in your community?

Examples

A) Tanya was hired as a lifeguard over the summer months and is in charge of watching the diving pool. She noticed that the divers were diving in a distinct pattern. The first person dove straight in, the second did a backward dive, the third performed a cannonball and the fourth person did a twist dive.

If this pattern continues, what type of dive will the thirteenth and twentieth person perform?

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20

The thirteenth diver will dive straight in, the twentieth diver will do a twist dive.

B) Which subject is missing from each of the shaded areas in the timetable?

Period	Day 01	Day 02	Day 03	Day 04	Day 05	Day 06
1	Math	Social	Math		Math	Social
2	Lang. Arts	Lang. Arts	Science	Science	Lang. Arts	
3	Science		Lang. Arts		Science	Science
4	Foods	Drama		Drama		
5		Foods	Drama	Foods	Drama	Foods

The missing subjects can be determined by looking for a pattern within the information.

In the timetable:

- Period 1 – mathematics and social studies form a pattern
- Periods 2/3 – language arts and science form a pattern
- Periods 4/5 – food studies and drama form a pattern

The complete timetable looks like this.

Period	Day 01	Day 02	Day 03	Day 04	Day 05	Day 06
1	Math	Social	Math	Social	Math	Social
2	Lang. Arts	Lang. Arts	Science	Science	Lang. Arts	Lang. Arts
3	Science	Science	Lang. Arts	Lang. Arts	Science	Science
4	Foods	Drama	Foods	Drama	Foods	Drama
5	Drama	Foods	Drama	Foods	Drama	Foods

Methods for Identifying and Extending Patterns

Guess and Check

When looking for patterns:

Guess: Guess how to get from one part of a pattern to the next part
Check: Test the pattern.

If correct, use that pattern to complete the missing parts.
If incorrect, follow the guess and check method until the pattern is correct.

T-tables

Use T-tables to help determine and complete patterns.

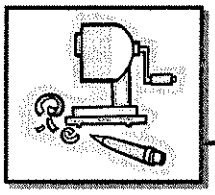
Example What are the y values for 6, 9 and 11?

x	y
2	5
3	6
5	8
6	
9	
11	

How does x relate to y?
Add 3 to the number in x to determine the value of y.

Solution: the values are 9, 12, 14

x	y
2	5
3	6
5	8
6	9
9	12
11	14



Practice: Identifying and Extending Patterns

1. What are the next three numbers in each pattern?

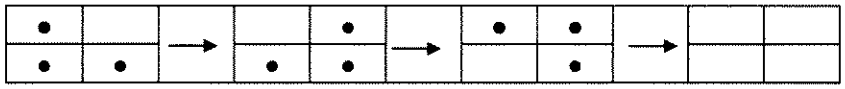
   a) 0, 2, 4, ____, ____, ____

   b) 3, 6, 9, ____, ____, ____

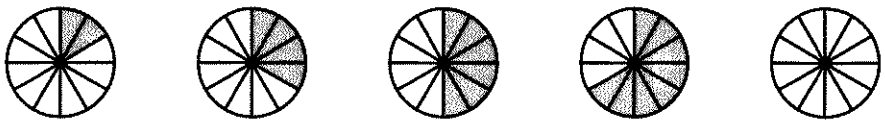
   c) 5, 7, 9, ____, ____, ____

   d) 8, 16, 24, ____, ____, ____

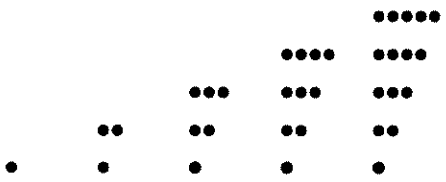
2. Complete the following pattern.



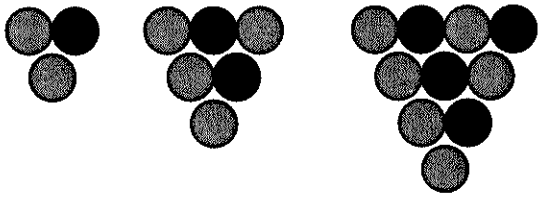
3. How many parts should be shaded in the last circle? How many parts would be shaded if another circle were drawn?



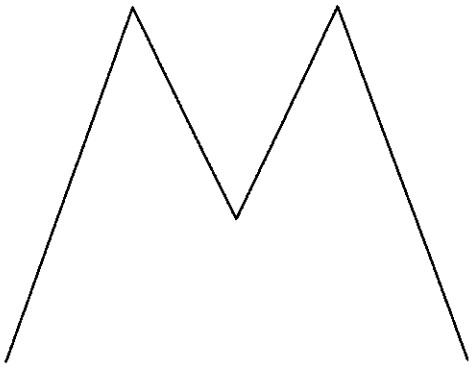
4. Identify the pattern below and discuss the next two or three in the series with a classmate.



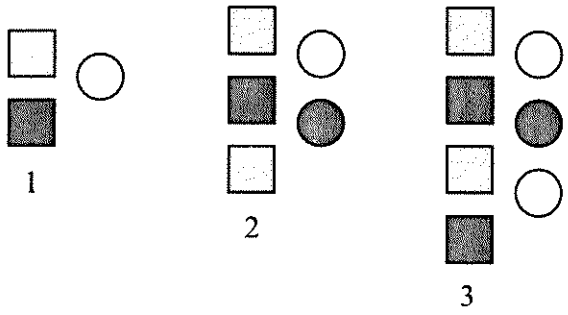
5. In each consecutive picture, another row of checkers is added—with one more checker than the previous row. The colours alternate—red, black, red, black. Describe or draw the next two pictures in the pattern.



6. Draw three straight lines on the figure below to form 9 triangles. (The triangles cannot overlap).



7. A new trend in stencilling is to paint different designs on each section of a fence. Look at the following fence design and complete the table below the pattern.



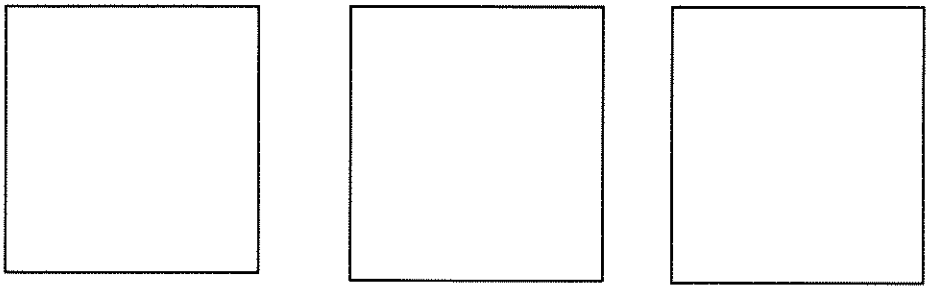
Fence Section	Squares	Circles
1	2	1
2		
3		
4		
5		
6		
7		

State the pattern in words _____

_____

_____

In the boxes below, draw the next 3 patterns (for fence sections 8, 9 and 10).



8 9 10

8. Gather representatives of coins or other manipulatives of various sizes. Have plenty of each type of coin (pennies, nickels, dimes, quarters) or object. Set up a pattern. Have a partner determine what your pattern is. Have your partner create the next two parts of your pattern.

9. Develop a variety of patterns using graph paper, the paint function of a software program or other methods. Challenge your classmates to determine the pattern.

10. Complete the following pattern. Use a calculator to verify your results.

$11 \times 10 = 110$
$11 \times 11 = 121$
$11 \times 12 = 132$
$11 \times 13 = 143$
$11 \times 14 =$ _____
$11 \times 15 =$ _____
$11 \times 16 =$ _____
$11 \times 17 =$ _____
$11 \times 18 =$ _____
$11 \times 19 = 209$
$11 \times 20 = 220$
$11 \times 21 = 231$
$11 \times 22 =$ _____
$11 \times 23 =$ _____

Note the pattern of the products. For example

$11 \times 12 = 1\ 3\ 2$

Add 1 + 2 to get the middle number 3.

$11 \times 13 = 1\ 4\ 3$

Add 1 + 3 to get 4.

Predict 11 × 22 and 11× 23.

Think of and describe other patterns with numbers, letters, words or in nature. For example:

$9 \times 1 = 9$
$9 \times 2 = 18, \quad 1 + 8 = 9$
$9 \times 3 = 27, \quad 2 + 7 = 9$
$9 \times 4 = 36, \quad 3 + 6 = 9$

11. Determine the relationships between numbers in the patterns below and complete the charts.

a)

x	2	3	4	5	6
y	6	9	12		

b)

x	3	5	6	8	9
y	10	16	19		

c)

x	0	1	4	8	10
y	0	5	20		

12. The table below shows money earned and money saved.

a) Determine the pattern and fill in the missing parts of the table.

Money Earned	$1.00	$2.00	$3.00	$4.00	$5.00	$6.00	$7.00	$8.00	$9.00
Money Saved	$0.25	$0.50				$1.50			$2.25

b) What is the pattern? How much money should be saved for each dollar spent? Describe the pattern in words.

c) Continue the pattern above, and fill in the missing parts of the table.

Money Earned	$12.00		$18.00		$25.00	$30.00	$35.00		$50.00
Money Saved		$3.75		$5.00				$12.00	

13. For every hour worked, Wu makes $6.50. Fill in the chart. (The first one has been done for you.)

Hours Worked	1	2	3	4	5	6
Wage Earned	$6.50					

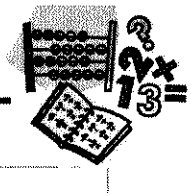
What would Wu earn working for 4.5h? What would he earn for working 8h?

14. Select a place of business or other source of information, and determine the hourly wage of an entry-level job.

Complete the table below based upon the hourly wage the employee makes.

Hours Worked	1	3	4	6	7	8
Wage Earned						

Using Equations to Describe Patterns



An equation shows a relationship between two or more unknowns (variables). Variables are letters used to represent known and unknown information. Variables help set up an equation.

Any letter can be used to represent numbers, such as a, b, n, x and y.

Some letters are not usually used because they can be mistaken for numbers or symbols:

- m may be mistaken for metres
- i may be mistaken for the number one
- o may be mistaken for a zero
- s may be mistaken for seconds.

Think of other letters that may be mistaken for numbers or symbols and discuss with a classmate or your teacher.

Example A group of friends take a six-day bicycle trip. They decide to travel 80 km every day to allow time for meals, rest and sightseeing.

The table below shows the kilometres travelled on the trip.

x = Day	1	2	3	4	5	6
y = Total Distance Travelled	80 km	? km	? km	? km	? km	? km

If they travelled 80 km each day, how far will they have travelled by day 3? Day 6?

Variables
x = the day of the trip
y = the total distance travelled

Set up an equation, the relationship between x and y:
80 kilometres are travelled each day
$y = x$ times 80 or $80x$
$y = x$ times 80 or $y = 80x$

When the relationship (or pattern) between numbers has been determined, predictions can be made.

Examples

A) In the table to the right, the pattern is y equals $2x + 1$.

Substituting numbers for x, y values can be determined.

x	$y = 2x + 1$
3	7
4	9
5	11
6	13
7	
8	
9	

If $x = 4$, solve for y
$y = 2 \times 4 + 1$
$y = 9$

If $x = 6$, solve for y
$y = 2 \times 6 + 1$
$y = 13$

B) Here is another example:

The missing values for y can be calculated:
Multiply the x value by 4 and subtract 1 from the product.

x	$y = 4x - 1$
2	7
4	15
6	23
8	21
10	39
12	47
14	
16	
18	

For example:

If $x = 9$, solve for y
$y = 4(9) - 1$
$y = 36 - 1$
$y = 35$

If $x = 20$, solve for y
$y = 4(20) - 1$
$y = 80 - 1$
$y = 79$

Extend the pattern by solving for y when x is 14, 16 and 18.

Write a pattern in words to help better understand a pattern and a problem.

Examples

A) The pattern to the right can be described:

x	$y = x + 3$
2	5
3	6
5	8
6	9
9	12
11	14

- take the number in the first column (x) and add 3 to get the number in the second column (y) OR $x + 3 = y$
- so, $y = x + 3$ This is called an equation and shows a relationship between the numbers in x and y.

B) Here is another example of a pattern that can be described using an equation.

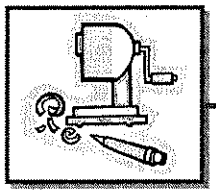
x	y
3	12
5	20
6	24
9	36
10	40
12	48

To get from x to y:

- take the number in x and multiply by 4 to get the number in y (OR $x \times 4 = y$)
- so, $y = 4x$ (or $4 \times x$, four multiplied by x)

When writing out number patterns in tables, always show the relationship in equation form.

x	$y = 4x$
3	12
5	20
6	24
9	36
10	40
12	48



Practice: Using Equations to Describe Patterns

1. Make a table using pencil and paper or a computer.

   Using the values 5, 7, 8, 9, 10, show the relationship $y = 2x$.

2. Monique purchased cards for her friends. The chart below shows how much she spent. x represents the number of cards she bought, y represents the amount of money the cards will cost. Write the equation to show the relationship and complete the chart.

x	$y =$
1	$1.10
2	$2.20
4	
6	
	$7.70
9	
	$11.00

3. Pedro accepted a position as an apprentice welder. Complete the table below to determine his wages. x represents the hours that Pedro worked. y represents the wages that he earned. Write the equation to solve for y.

x	$y =$
1	$16.50
3	
	$49.00
6	
7.5	
	$140.25
	$181.50

4. Stephen's parents are coordinating a family reunion and ask him to make his famous wonton soup. One batch of Stephen's soup will feed 3 people. Look at the table below and determine how many batches of soup Stephen will need to make for 45 people.

# of Batches	# of People
x	$y =$
1	3
2	6
4	
5	
	21
9	
	30
12	
	45

Extend this table to answer the following questions.

a) Write the equation to solve for y.

b) Use the equation and solve for y for 75 and 117 guests.

5. Use guess and check or another method to determine the patterns in the following T-tables. Indicate the relationship by completing the equation for y.

a)

x	$y =$
	3
2	6
4	12
6	
	21
9	
	33

b)

x	$y =$
3	10
5	12
10	17
15	
	25
21	
	38

c)

x	$y =$
9	6
10	7
12	9
14	
	12
16	
	16

d)

x	$y =$
2	8
4	16
5	20
7	
	36
10	
	48

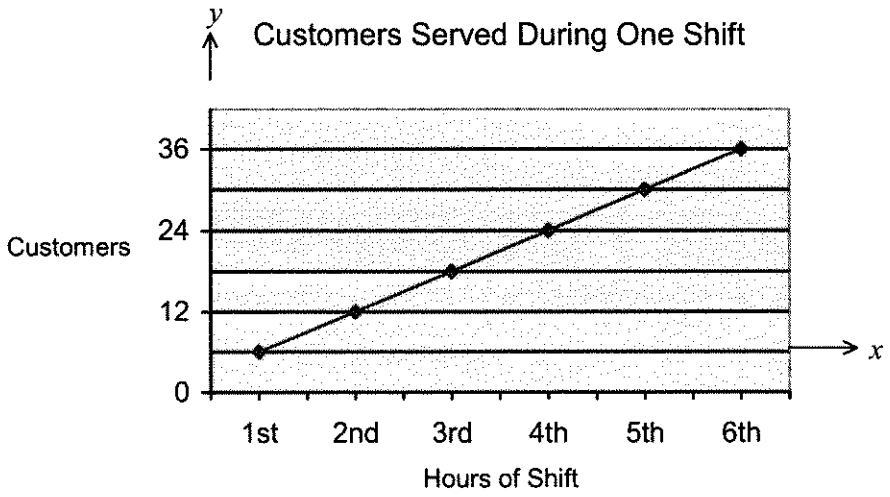
e)

x	$y =$
1	3
2	5
4	9
7	
	19
10	
	23
	27

f)

x	$y =$
1	1
3	5
4	7
6	
	15
9	
	19
	23

6. a) Write an equation solving for y to show the relationship between the hours of the shift and the number of customers served.

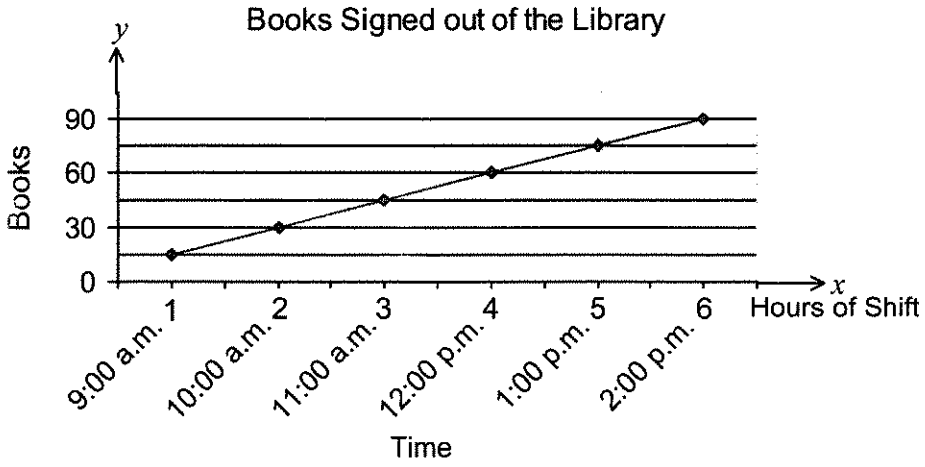


b) Complete the table below.

x	$y =$
1	6
2	
3	
4	
5	
6	

c) In word form, state the relationship between the variables x and y.

7. a) Write an equation solving for y to show the relationship between the hours of the shift and the number of books signed out.



b) Complete the table below.

x	$y =$
hour 1	15
hour 2	
hour 3	
hour 4	
hour 5	
hour 6	

c) In word form, state the relationship between the variables x and y.

8. The T-table shows the relationship between the cups of shredded carrots and the cups of shredded cabbage a caterer uses when making coleslaw.

x	$y =$
2	6
5	9
6	
8	
9	
11	15
12	

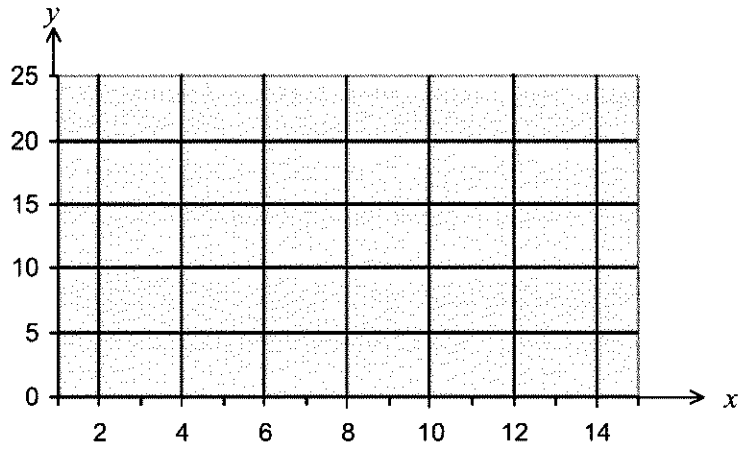
x = cups of carrots
y = cups of cabbage

a) Determine the pattern, write the equation and complete the T-table.

b) List the ordered pairs from the table above:

(2, 6) (5, 9) (_____, _____) (_____, _____) (_____, _____) (_____, _____) (_____, _____)

c) Place the ordered pairs on the graph below:



d) Extend the pattern:

Predict the y value if x is 15.

Write the above as an ordered pair. _____

In word form, explain the pattern/relationship between the x and y variables.

_____

9. The T-table shows the relationship between the amount of time Wendy takes to weld and the centimetres of pipe welded.

a) Complete the T-table and write the equation to solve for y.

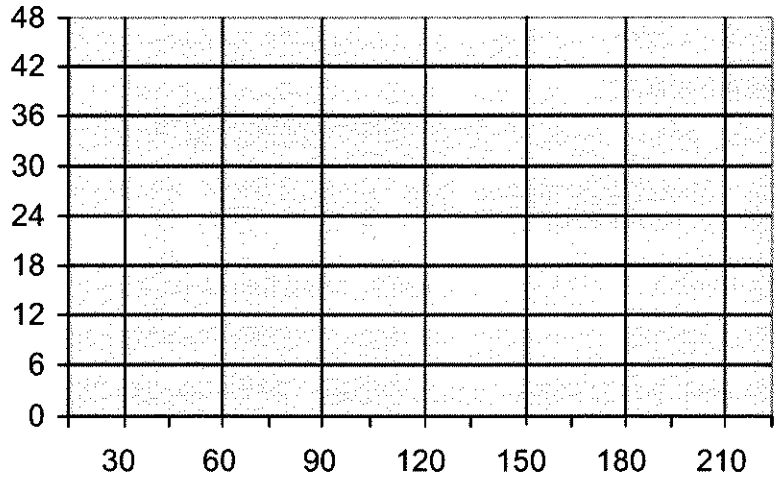
x (in seconds)	y (in cm)
30	2
60	4
90	
120	
150	
180	
210	14

x = Time (in seconds)
y = Centimetres of pipe welded

b) List the ordered pairs from the table above:

(30, 2) (60, 4) (____, ____) (____, ____) (____, ____) (____, ____) (210, 14)

c) Place the ordered pairs and headings on the graph below:



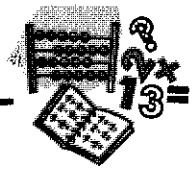
d) Extend the pattern:

Predict how much pipe can be welded in 4 minutes. _____

Write the above as an ordered pair. _____

Use word form to explain the pattern/relationship between the x and y variables. _____

Examining Equality



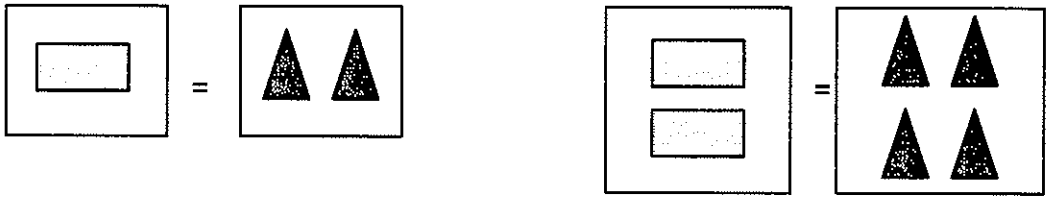
Shapes and patterns can demonstrate relationships.

Examples

A) If ... **Then ...**

Two squares equals four circles.
So, one square equals two circles.
If there were five squares, how many circles would there be?

B) If ... **Then ...**



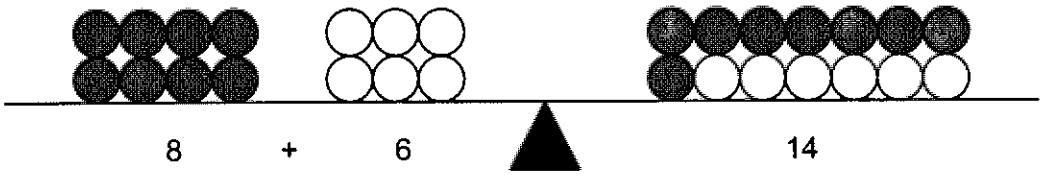
One rectangle equals two triangles.
So, two rectangles equal four triangles.
If there were three rectangles, how many triangles would there be?

In number equations, the equal sign is the same as the fulcrum (▲) of a balance. The amount to the left of the equal sign must equal the amount to the right of the equal sign to balance the equation.

Example 8 + 6 = 14

The amount on both sides of the equal sign is 14.
The sides are equal to each other.

The diagram shows the relationship 8 + 6 = 14. The fulcrum represents the equal sign.



Proving Equality

Example

A) 6 × 4 = 3 × 8

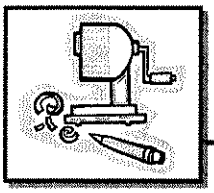
To prove that the equation above is true, solve each side of the equation.
6 × 4 = 24
3 × 8 = 24
24 = 24
The equation is true.

B) 5 × 7 = 10 × 3

5 × 7 = 35
10 × 3 = 30
35 ≠ 30
The equation is not true.

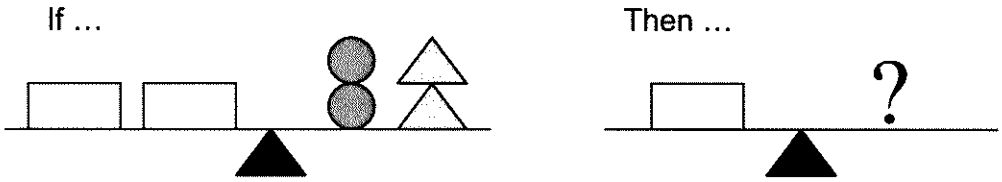
To state that an equation is not true, use the "does not equal" sign: ≠

5 × 7 ≠ 10 × 3

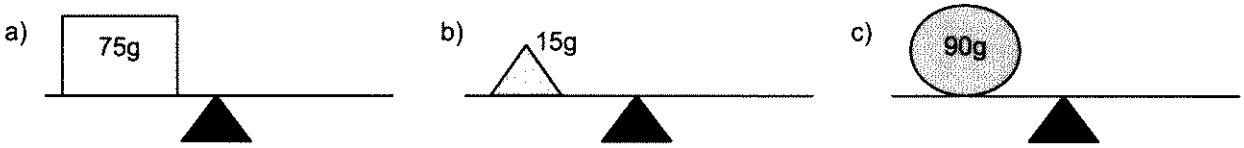


Practice: Examining Equality

1. What shapes are needed to balance the scale below? Discuss your response with your classmates.



2. Use the figures in the box below to balance the following. Compare your answers with classmates.

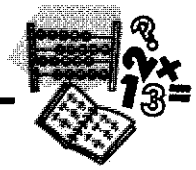


3. Prove whether or not the following equations are true.

a) $6 \times 6 = 5 \times 7 + 1$
b) $7 \times 3 = 20 + 4$
c) $12 - 6 = 2 \times 3$
d) $18 \div 6 = 9 \div 3$

Compare your responses with those of classmates or your teacher.

Solving Equations



Solving an equation means figuring out what value the variable represents. Recall that variables are letters or symbols used to represent unknown numbers.

There are several different ways to solve equations, including the guess and check method, and paper and pencil method.

Examples

A) $7 + n = 12$

Guess and check method:

$7 + 0 = 7$
$7 + 1 = 8$
$7 + 2 = 9$
$7 + 3 = 10$
$7 + 4 = 11$
$7 + 5 = 12$

Paper and pencil method:

$7 + n = 12$
$n = 12 - 7$
$n = 5$

B) Mike and a friend are getting together for a movie marathon weekend. Mike has coupons to rent 8 free movies, and together they have enough coupons to rent 13 movies. How many coupons does Mike's friend have?

Set up an equation: $8 + n = 13$. Solve for n.

Guess and Check Method:

$8 + n = 13$ ⟶ Think: $8 + 1 = 9$
$8 + 2 = 10$
$8 + 3 = 11$
$8 + 4 = 12$
$8 + 5 = 13$

$8 + 5 = 13$, $n = 5$

Pencil/Paper Method:

$8 + n = 13$
$8 - 8 + n = 13 - 8$
$n = 13 - 8$
$n = 5$ Check: $8 + 5 = 13$, $13 = 13$

The steps for solving different kinds of equations using the paper and pencil method are described on the following pages.

Solving Type 1 Equations—Addition

Goal: find the missing number (x) in the equation.

To figure out what the missing number is, you have to change the equation so it ends up looking like:

x = number

To do that, get rid of all of the numbers on the left side except one x:

Example $x + 8 = 33$ is the same as writing ? + 8 = 33 or □ + 8 = 33

To get rid of a number that is added, just do the opposite—subtract it again!

$$x + 8 = 33$$
$$\underline{-8 = -8}$$

Remember, just like in fractions, whatever you do on one side of the = sign, you have to do on the other side.

$$x + \cancel{8} = 33$$
$$\underline{-\cancel{8} = -8}$$
$$25$$

On the left side, the numbers cancel themselves out, leaving only one x—that's your goal!

$$x = 25$$

Check your Work

Put the missing number (what x is) into the original equation and solve it – make sure the answer is the one they gave you in the first equation.

$$x + 8 = 33$$
$$25 + 8 = 33 \ \checkmark$$

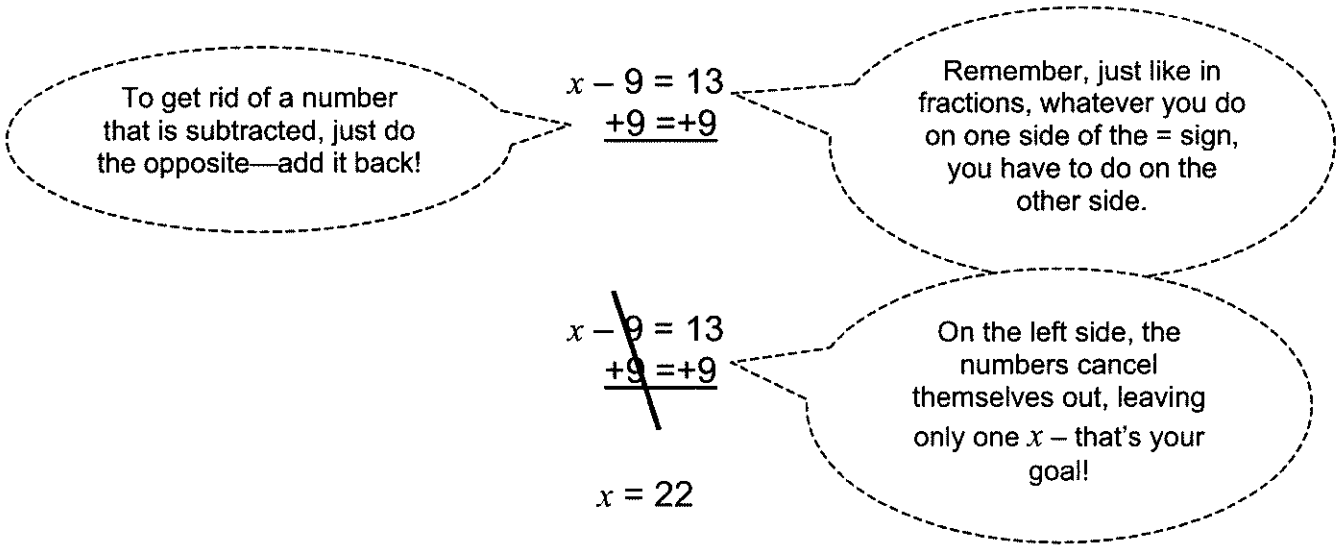
Solving Type 1 Equations—Subtraction

Goal: find the missing number (x) in the equation. To figure out what the missing number is, you have to change the equation so it ends up looking like:

x = number

To do that, get rid of all of the numbers on the left side except one x:

Example $x - 9 = 13$ is the same as writing $? - 9 = 13$ or $\square - 9 = 13$



Check your Work

Put the missing number (what x is) into the original equation and solve it – make sure the answer is the one they gave you in the first equation.

$$x - 9 = 13$$
$$22 - 9 = 13 \ \checkmark$$

Type 1 Equations—Multiplication

Goal: find the missing number (x) in the equation.

$6x = 24$ is the same as writing ? × 6 = 24 or □ × 6 = 24

To figure out what the missing number is, you have to change the equation so it ends up looking like:

x = number

Example To do that, get rid of all of the numbers on the left side except one x:

$$\frac{6x}{6} = \frac{24}{6}$$

To get rid of a number that is multiplied, just do the opposite—divide it again!

Remember, just like in fractions, whatever you do on one side of the = sign, you have to do on the other side.

$$\frac{\cancel{6}x}{\cancel{6}} = \frac{24}{6}$$

On the left side, the numbers cancel themselves out, leaving only one x—that's your goal!

$$x = 4$$

Check your Work

Put the missing number (what x is) into the original equation and solve it – make sure the answer is the one they gave you in the first equation.

$$6x = 24$$

$$6(4) = 24 \checkmark$$

Type 1 Equations—Division

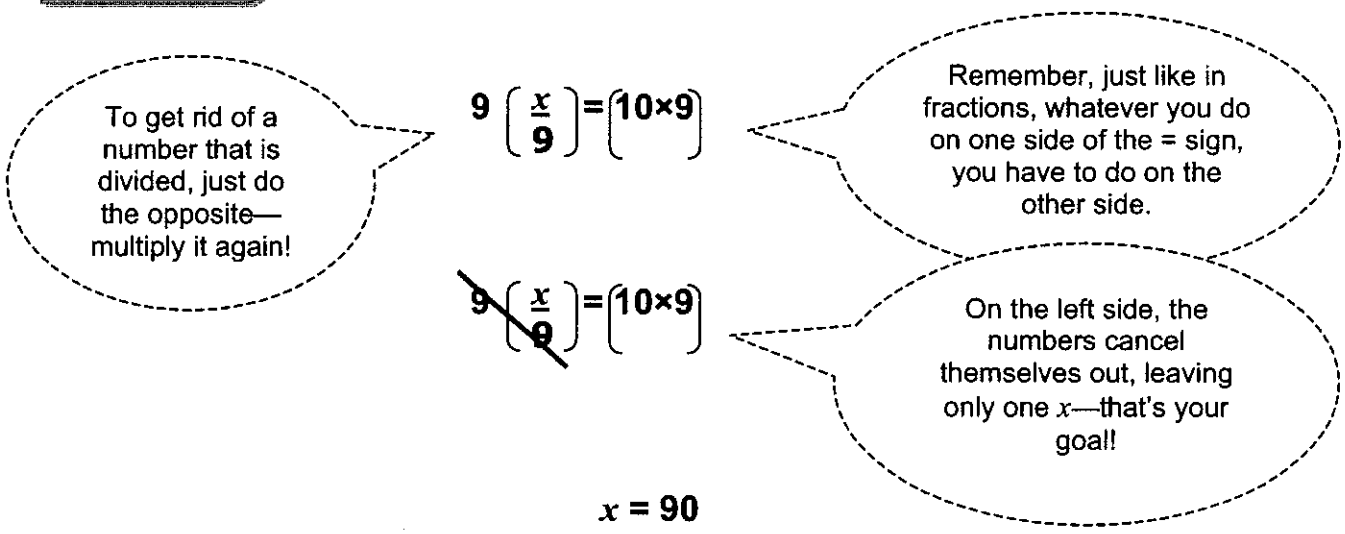
Goal: find the missing number (x) in the equation.

$\frac{x}{9} = 10$ is the same as writing ? 9 = 10 or □ 9 = 10

To figure out what the missing number is, you have to change the equation so it ends up looking like:

x = number

Example To do that, get rid of all of the numbers on the left side except one x:



Check your Work

Put the missing number (what x is) into the original equation and solve it – make sure the answer is the one they gave you in the first equation.

$$\frac{x}{9} = 10$$

$$\frac{90}{9} = 10 \checkmark$$

Type 2 Equations

Goal: find out what the missing number (x) in the equation is.

The goal is the same as it was for type one equations, but there is one more step to do to get the answer. The good news is, after that first extra step, the equation becomes just like a type one again.

So ... type 2 equations become type 1 equations, and you already know how to do those!

$5x - 7 = 8$

Notice that this equation involves multiplication and subtraction. That's why there's an extra step.

To figure out what the missing number is, you have to change the equation so it ends up looking like:

x = number

Example To do that, get rid of all of the numbers on the left side except one x:

Start with the "regular" numbers (those without the x) because they're just easier!

$5x - 7 = 8$
$+7 \quad +7$

Remember, just like in fractions, whatever you do on one side of the = sign, you have to do on the other side.

$5x - \cancel{7} = 8$
$\cancel{+7} \quad +7$

On the left side, the numbers cancel themselves out, leaving $5x$—you're getting close!

Now we're back to a type one equation—phew!

$5x = 15$

$\frac{5x}{5} = \frac{15}{5}$

$x = 3$

Finished!

Check:

$5x - 7 = 8$

$5(3) - 7 = 8$

$15 - 7 = 8$ ✓

Example

$$\frac{x}{8} + 3 = 12$$

Notice—this equation involves division and addition and that's why there's an extra step.

Get rid of all of the numbers on the left side except one x:

Start with the "regular" numbers (those without the x) because they're just easier!

$$\frac{x}{8} + 3 = 12$$

Remember, just like in fractions, whatever you do on one side of the = sign, you have to do on the other side.

$$\frac{x}{8} + \cancel{3} = 12$$
$$\quad\ -3 \quad -3$$

On the left side, the numbers cancel themselves out, leaving x divided by 8—you're getting close!

Now we're back to a type one equation—phew!

$$\frac{x}{8} = 9$$

$$8\left(\frac{x}{8}\right) = 9(8)$$

$$x = 72$$

Finished!

Check:

$$\frac{x}{8} + 3 = 12$$

$$\frac{72}{8} + 3 = 12$$

$$9 + 3 = 12 \ \checkmark$$

Type 3 Equations

Goal: find the missing number (x) in the equation.

The goal is the same as it was for type one and two equations, but there is one more step to do to get the answer.

So ... type 3 equations eventually become type 1 equations, and you already know how to do those!

$4x + 3x = 7$

Notice—this equation has more than one number with an *x* beside it and that's why there's an extra step.

To figure out what the missing number is, you have to change the equation so it ends up looking like:

x = number

Begin by figuring out how many xs there are altogether:

Example

$4x + 3x = 7$

Add up all of the x's.

$7x = 7$

Now we're back to a type one equation—phew!

$\frac{7x}{7} = \frac{7}{7}$

$x = 1$

Finished!

Check:

$4x + 3x = 7$

$4(1) + 3(1) = 7$

$4 + 3 = 7$ ✓

Example

$$9x + 2 = 5x + 30$$

Notice—this equation has numbers with x's on both sides of the = sign and that's why there's an extra step.

Begin by getting all of the xs on the same side of the = sign:

$$\begin{array}{r} 9x + 2 = 5x + 30 \\ -5x \qquad\quad -5x \\ \hline 4x \end{array}$$

Remember, just like in fractions, whatever you do on one side of the = sign, you have to do the same on the other side.

On the right side, the numbers cancel themselves out, leaving x divided by 8—you're getting close!

Now we're back to a type two equation—phew!

$$\begin{array}{r} 4x + 2 = 30 \\ -2 \quad -2 \\ \hline 28 \end{array}$$

Now we're back to a type one equation—phew!

$$\frac{4x}{4} = \frac{28}{4}$$

$$x = 7$$

Finished!

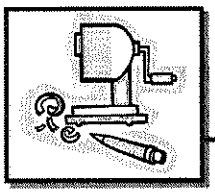
Check:

$9x + 2 = 5x + 30$

$9(7) + 2 = 5(7) + 30$

$63 + 2 = 35 + 30$

$65 = 65$ ✓



Practice: Solving Equations

1. Use the guess and check method and the paper/pencil method to solve the following.

   Harnaik gets 100 free minutes of Internet time per month.
   He has already used 64 minutes.
   How many minutes does he have left?

   Write out an equation: _____

Guess and Check Method

Think: 64 + 1 = _____ (your answer is way off—pick a larger number)
64 + 10 = _____ (you are still way off)
64 + 25 = _____ (you are getting closer)
64 + 35 = _____ (very close)
64 + __ = _____

64 + _____ = 100 Harnaik has _____ minutes of free Internet time left.

Pencil/Paper Method

Known information:
Harnaik gets 100 minutes of Internet time.
He has used 64 minutes.

What is being asked?
How much Internet time does Harnaik have left?

Let m be Minutes he has left.

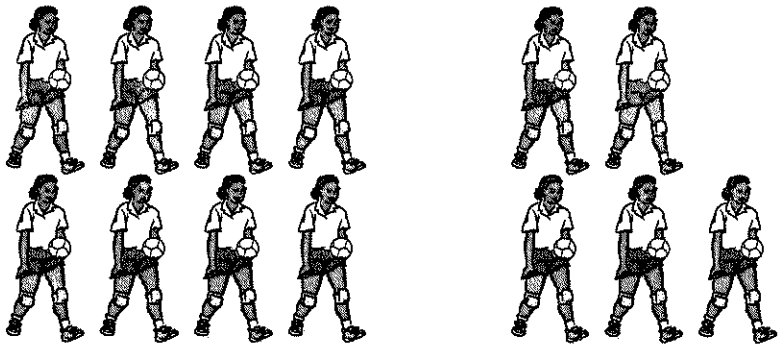
$64 + m = 100$
$64 - \_\_\_\_\_ + m = 100 - \_\_\_\_\_$
$m = 100 - \_\_\_\_\_$
$m = \_\_\_\_\_$

Harnaik has _____ minutes of free Internet time left.

2. Jules is helping organize a volleyball tournament. He calls 5 of his friends and has 8 friends left to call. How many friends did Jules call in total?

Let  = the number of friends Jules called in total.

Solve for  .



n 

 $= \quad 8 \quad + \quad 5$

Jules called _____ friends.

3. Balance the equations using a variety of methods, such as pencil and paper, and diagrams.

a) _____ + 6 = 13

b) 7 + _____ = 17

c) 2 + 9 = _____

d) _____ + 12 = 15

e) 14 – 9 = _____

f) _____ – 8 = 3

g) 13 – _____ = 6

h) 20 – 8 = _____

i) 9 × 2 = 6 × _____

j) 4 × 8 = 3 × ____ + 2

k) 30 ÷ 6 = 20 – _____

l) 56 ÷ 7 = ____ ÷ 2

4. Solve for x in the following equations.

a) $x + 5 = 24$ $x =$ b) $4 + x = 19$ $x =$

c) $x - 10 = 12$ $x =$ d) $32 - x = 15$ $x =$

e) $x + 7 = 34$ $x =$ f) $x - 14 = 3$ $x =$

g) $x + 45 = 126$ $x =$ h) $37 + x = 213$ $x =$

i) $x - 47 = 56$ $x =$ j) $327 - x = 68$ $x =$

k) $x + 239 = 547$ $x =$ l) $x - 159 = 218$ $x =$

5. Solve for x in the following equations.

a) $4x = 36$ $x =$ b) $2x = 56$ $x =$

c) $3x = 21$ $x =$ d) $8x = 56$ $x =$

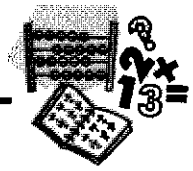
e) $12x = 144$ $x =$ f) $15x = 210$ $x =$

g) $5x - 7 = 8$ $x =$ h) $4x - 6 = 38$ $x =$

i) $3x + 9 = 27$ $x =$ j) $7x + 12 = 47$ $x =$

k) $21 + 6x = 75$ $x =$ l) $8x - 24 = 136$ $x =$

Variables and Constants



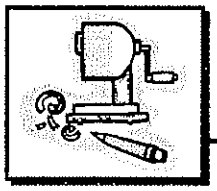
You already know that **variables** are letters used to represent known and unknown information in an equation.

Some equations also include **constants**. These are things that do not change or change very little over long periods of time.

Examples

Some familiar constants include:

- Wednesday follows Tuesday
- the speed of the earth in orbit
- the distance between planets
- the value of π is 3.14 (approximately)
- the speed of light is 3 × 108 m/s.



Practice: Identifying Variables and Constants

1. Examine the relationship between the circumference and diameter of a circle. What is the constant?

2. Einstein's law of relativity states that $E = mc^2$. What do the variables represent? Which is the constant?

3. Look at the table below. Place a checkmark in the appropriate box to show whether each statement contains variables or constants.

Statements	Variable	Constant
The radius of a circle is 4 cm.		
Water boils at 100°C.		
There are 24 hours in a day.		
10 millimetres equals 1 centimetre		
You get back $2.38 change when you pay with a twenty-dollar bill.		
There are 126 pages in a book.		
100 divided by 10 equals 10.		
Plants grow 2.5 centimetres per week.		
3 teaspoons equal 1 tablespoon.		
Tacos cost $1.29.		
There are 7 days in a week in Canada.		
The relationship between the circumference of a circle and its diameter is pi.		

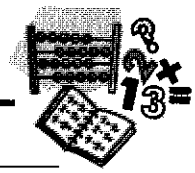
Identify three more constants.

1. _____
2. _____
3. _____

Identify three more variables.

1. _____
2. _____
3. _____

Writing Mathematical Expressions



Algebra is using equations to solve problems.

Algebraic equations have an unknown, which is information that must be solved for. The unknown number is represented by a letter. This letter is called a **variable**.

Assign **variables** (letters) to represent the unknown information and set up an equation to solve for the unknown.

Examples

A)

Statement/Word Problem	Expression
A number increased by 8	$n + 8$
12 decreased by a number	$12 - n$
Your age times 2	$2a$
Your money increased by $25.00	$m + 25$
The number of workers decreased by 7	$w - 7$
A number divided by 5	$n \div 5$
The price of an item plus $3.21 sales tax	$p + \$3.21$
$5.00 off the lowest sale price	$n - \$5.00$
Seven times the length of the window	$7 \times l$
Your regular pay plus a $50.00 bonus	$p + \$50.00$
Regular price minus a $9.00 late fee	$r - \$9.00$

B) Pascal is doing a research project on careers and is using 12 books from home. He signs out 4 books from the library. How many books does Pascal have in all to help him with his project?

Known information:

Pascal is using 12 books from home.
Pascal signed out 4 books from the library.

What needs to be solved?

How many books is Pascal using?

Let n be the total number of books.
Solve for n by setting up an equation.

Procedure	Example
1. Create a number sentence.	$n = 12 + 4$
2. Solve for n.	$n = 16$
3. Write a summary statement.	Pascal has a total of 16 books.

C) Keisha is working on a science project. She has some books signed out and returns 5 books to the library. She has 6 books left. How many books did Keisha originally have?

Known information:
Keisha returned 5 books.
She now has 6 books.

What needs to be solved?
How many books did Keisha originally have?

Let n be the number of books Keisha originally had.
Set up the equation $n - 5 = 6$
Solve for n.
Isolate the unknown on the left by using opposite operations.

$$n - 5 + 5 = 6 + 5$$
$$n = 6 + 5$$
$$n = 11$$

Keisha originally had 11 books signed out.

D) For his birthday, Keith received 7 CDs. Now he has 15 CDs. How many CDs did Keith originally have?

Let n be the number of CDs Keith originally had. Solve for n.

$n = 15 - 7$ or $n + 7 = 15$
$n = 8$

Keith originally had 8 CDs.

E) Sami spent $6.00 on lunch. He has $13.00 left. How much money did Sami have before lunch?

Let n be the money that Sami had before lunch. Solve for n.
n = $13.00 + $6.00 or n – $6.00 = $13.00
n = $19.00

Sami had $19.00 before lunch.

F) A T-shirt is on sale for $8.00 off the regular price. The regular price is $18.00. What will it cost?

The regular price is $18.00.
The sale price is $8.00 off the regular price.

Let n be the cost of the shirt on sale.
Solve for n.

n = $18.00 – $8.00
n = $10.00
The T-shirt will cost $10.00.

G) A CD costs $20.99. Jerome pays $4.00 for shipping and handling. What will be the total cost?

Let n be the total cost.
Solve for n.

n = $20.99 + $4.00
n = $24.99
The total cost of Jerome's order will be $24.99.

H) Megan received a bonus of 3 times her regular pay. Her regular pay is $45.00. How much is Megan's bonus?

Let n be her bonus.
n = $45.00 × 3
n = $135.00
Megan's bonus is $135.00.

I) Akmed is working at the door at a school dance. He sells 254 tickets in total, 136 to males. How many females purchased tickets?

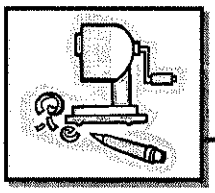
Let F = the number of females who bought tickets.
$F + 136 = 254$
OR
$F = 254 - 136$
$F = 118$
118 females bought tickets.

J) The school library has 2467 books in total and 1763 are fiction books. How many are nonfiction?

Let N = the number of nonfiction books.
$N + 1763 = 2467$
OR
$N = 2467 - 1763$
$N = 704$
704 books are nonfiction.

K) 10 468 people attended an Oilers/Flames game. There are 249 Flames' fans.

O = the number of Oilers' fans
$O + 249 = 10\,468$
OR
$O = 10\,468 - 249$
$O = 10\,219$
There were 10 219 Oilers' fans.



Practice: Writing Mathematical Expressions

1. Write expressions for the following.

   a) Height increased by 7 cm

   b) An art mark decreased by 12%

   c) Hourly wages cut in half

   d) The number of friends increased by 10

   e) An English mark three times greater than a drama mark

2. Select an employer or other representative of a local business. With approval, ask questions about the workplace. Use the information to set up equations like the following.

   a) The number of male and female workers
   b) The number of full-time and part-time employees
   c) The number of people in management positions and regular positions.

3. Look around your school. Create mathematical expressions for the following.

   a) The number of teachers and students
   b) The number of computers and telephones
   c) The number of classrooms and exit doors
   d) The number of students and water fountains
   e) The number of computers and students.

   Compare and discuss your expressions with a classmate or your teacher.

4. Create word problems for the following expressions and share them with a classmate.

   E.g., $16 + 2 = n$ Ben is two years older than his sister who is 16. How old is Ben?

   a) $2 + 7 = n$

   b) $3n = 6$

   c) $n - 8 = 12$

   d) $n + 6 = 14$

5. Write equations using a variable for each of the following word problems and solve them.

   a) Pierre paid $35.00 for a new T-shirt and jeans. The price of the T-shirt was $9.00. How much were the jeans?

   b) Jane owes her two friends a total of $153.00. She owes Jim $47.00. How much does she owe Charlene?

   c) Ben needs 541 bottles to beat his record for last year's bottle drive. If he collected 315 bottles in the morning, how many must he collect in the afternoon to beat his record?

   d) Chen drove a total of 632 km over three days. On the first day he drove 217 km. On the third day he drove 126 km. How many kilometres did he drive on the second day?

6. Assign variables to the unknowns and set up an equation for each of the following. Solve for the unknown.

   a) There are 378 cats and dogs at the SPCA; 234 are cats.

   b) You have 23 pairs of black and white socks in your dresser drawer; 8 pairs are black.

   c) There are 126 cars and trucks in the school parking lot; 28 are trucks.

   d) Sandy was keeping track of students entering the school dance. When she did her tally, she found that 137 people were in the gym. If 71 of these people were girls, how many boys were in the gym?

7. Write equations using a variable for each of the following word problems and solve them.

   a) Tina spent a total of $80.00. She bought a dress for $44.00 and 3 T-shirts. How much did each T-shirt cost?

   b) Harry spent a total of $55.00 at the fair. He bought lunch for $10.00 and went on different rides that each cost $1.50. How many rides did Harry go on?

   c) Bobbi Lyn earned a total of $125.00 at the horse fair. She won 3 different competitions, but the entry fee for the fair was $25.00. How much did Bobbi Lyn win for each competition?

8. The following table shows the medal results from the city high school track meet.

High School	Gold Medals	Silver Medals	Bronze Medals	Total Medals Won
Franklin	7	6	5	18
Belmore		2	0	14
Shelby	1		4	7
Avondale	3		4	9
Delmead		6	8	22

Using the information from the table above, create equations and solve for the unknowns. The first one has been done for you.

Write an equation to help you determine how many bronze medals Franklin won.

Let B be the number of bronze medals.
$7 + 6 + B = 18$
Solve for B.
$13 + B = 18$
$13 - 13 + B = 18 - 13$
$\boldsymbol{B = 5}$
Therefore, Franklin won 5 bronze medals.

a) Write an equation to determine how many gold medals Belmore won.

b) Calculate the number of gold medals won by Belmore. Show your work and record your answer in the table.

c) Write an equation to determine how many silver medals Shelby won.

d) Calculate the number of silver medals won by Shelby. Show your work and record your answer in the table.

e) Write an equation to determine how many silver medals Avondale won.

f) Calculate the number of silver medals won by Avondale. Show your work and record your answer in the table.

g) Write an equation to determine how many gold medals Delmead won.

h) Calculate the number of gold medals won by Delmead. Show your work and record your answer in the table.

9. In the box below, create a word problem for a classmate to solve using an equation.