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# Math 9 K&E

## Integers

<u>Integers</u>	
-Numberlines	-Integer chips
 =+1	Positive and
 =-1	Negative Numbers



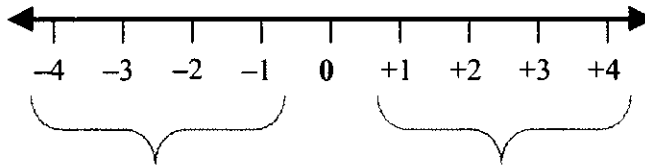
# Positive and Negative Numbers



**Integers** are positive and negative numbers. They are shown with a positive or negative sign.

Integers are always compared to zero. For example,  $-2$  (2 less than zero),  $+6$  (six more than zero).

Integers can be represented on a number line as follows:



Numbers to the left of zero are **negative** numbers.

Numbers to the right of zero are **positive** numbers.

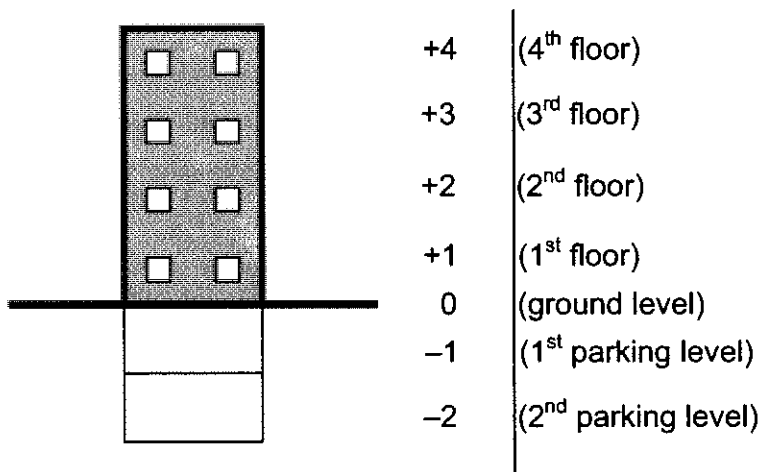
A  $-$  sign is used to represent negative numbers.

A  $+$  sign is used to represent positive numbers.

## Examples

$-2$  is negative two.  
 $+3$  is positive three.

Check out how integers are used to show the floors of the building in the example below.



Integers are used in lots of different ways.

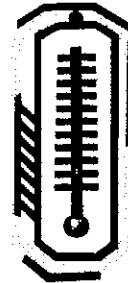
## Temperature

Integers are used to describe how warm or cold the temperature is.

### Examples

+30°C is a very warm day.

-28°C is a very cold day.



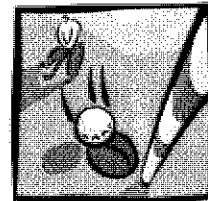
## Sports

Integers are used to describe golf scores.

### Examples

+2 represents two strokes above par.

-1 represents one stroke below par.



## Depth and Elevation

Integers are used to describe movement upwards (elevation) or downwards (depth) in relation to sea level.

### Examples

+200 m would represent 200 m above sea level.

-20 m would represent 20 m below sea level.



## Bank Deposits/Withdrawals

Integers are used to describe if money is added to or taken out of an account.

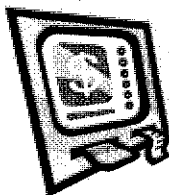
### Examples

+\$200.00

may appear on your bank statement if \$200.00 was added to your account.

-\$200.00

may appear on your bank statement if \$200.00 was withdrawn from your account.



When money is earned, it can be represented by a positive sign (+). When money is spent, it can be represented by a negative sign (-).

The chart below displays common vocabulary used when earning and spending money.

<b>Earn Money</b>	<b>Spend Money</b>
Positive Gain Above Plus Increase More	Negative Loss Below Minus Decrease Less
+	-

**Example**



Claire had a busy week! She received her weekly allowance of \$15.00, got a haircut for \$10.00, earned \$22.00 baby-sitting, received \$90.00 in birthday money and bought a new jacket for \$30.00.

Each of Claire's money transactions represents a gain (+) or loss (-) and are represented as follows.

- +\$15.00 allowance
- \$10.00 haircut
- +\$22.00 baby-sitting
- +\$90.00 birthday money
- \$30.00 new jacket



## Practice: Converting Positive and Negative Numbers

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Convert each of the following statements into integers using the correct sign in front of each number.

1. An increase of ten points.
2. A loss of six points.
3. A gain of fifty dollars.
4. A decrease in temperature by four degrees Celsius.
5. A loss of two hundred dollars.
6. A depth of thirty-five metres below the surface.
7. A climb of one hundred sixty-five metres.
8. Sixteen degrees above zero degrees Celsius.
9. A withdrawal of two thousand five hundred fifty-five dollars.
10. Seven degrees Celsius above zero.



## Practice: Earnings and Integers

11. It has been a busy week for Kiran! She has been earning money by baby-sitting every day. Every night, she has been out with her friends. Look at Kiran's activity list below. Place each transaction in the bankbook to determine her final balance.

July 07  
 Baby-sat – earned \$25.00  
 Picked up new book – spent \$10.62  
 Went to movies – spent \$15.75

July 08  
 Bought slurpees for kids – spent \$4.45  
 Paid sister back – spent \$15.00

July 09  
 Baby-sat – earned \$31.25  
 Bought Dad a sundae – spent \$3.76

July 10  
 Met friends for breakfast – spent \$8.97  
 Bought a new discman – spent \$76.98

July 11  
 Bought donuts for kids – spent \$4.86  
 Baby-sat – earned \$48.35

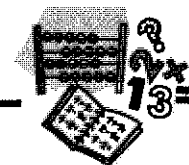
Minus or add the amount of the transaction from the total in this column. ↓

Date	Transaction	Expenses	Deposit	Balance
				+\$89.03
July 07	Baby-sitting Deposit		+\$25.00	+\$114.03
July 07	Bookstore	-\$10.62		+\$103.41
July 07	Movie Theatre	-\$15.75		+\$87.66
July 08				
July 08				
July 09				
July 09				
July 10				
July 10				
July 11				
July 11				





## Comparing Positive and Negative Numbers

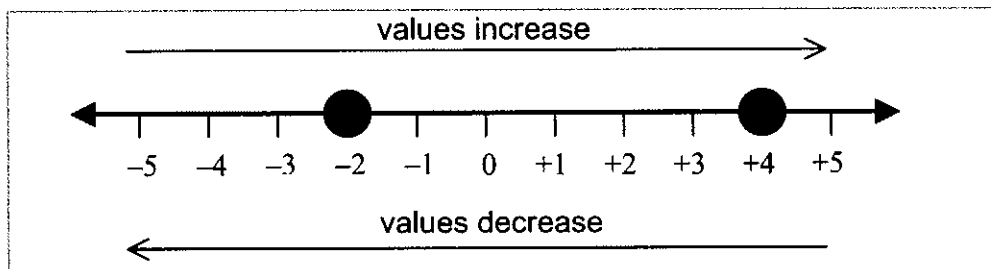


Integers can be illustrated and compared in a variety of ways, such as a number line, thermometer or grid paper.

### Comparing Numbers Using a Number Line

**Example** Compare  $-2$  and  $+4$ .

Place a dot on the number line to represent and compare  $-2$  and  $+4$ .

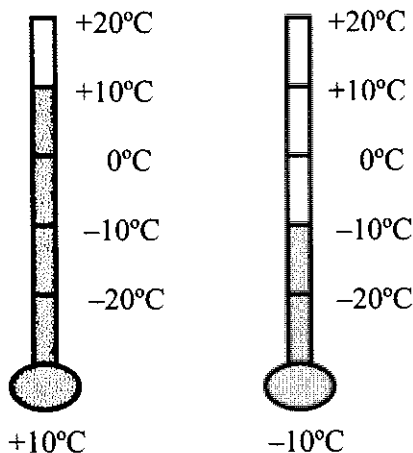


The number line shows that  $-2$  is less than ( $<$ )  $+4$ .

### Comparing Numbers Using a Thermometer

Thermometers are like number lines that are vertical (up/down) instead of horizontal (left/right).

**Example** Compare  $+10^{\circ}\text{C}$  and  $-10^{\circ}\text{C}$ .



The higher the level of liquid in the thermometer, the greater the temperature.

The lower the level of liquid in the thermometer, the lower the temperature.

Therefore,  $+10^{\circ}\text{C}$  is more than ( $>$ )  $-10^{\circ}\text{C}$ .

## Comparing Numbers Using Grid Paper

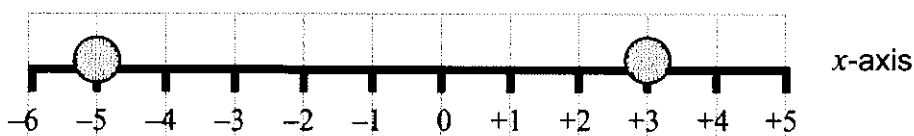
Grid paper can be used for plotting integers as ordered pairs.

Grid paper can be used to represent a number line to horizontally compare numbers, similar to the  $x$ -axis of the coordinate plane.

To compare numbers using grid paper, select a location along a central grid line to represent zero, and label the number line.

### Example

Compare  $+3$  and  $-5$ . Which has the greater value?



Since  $+3$  is further to the right on the number line,  $+3$  is greater than ( $>$ )  $-5$ .

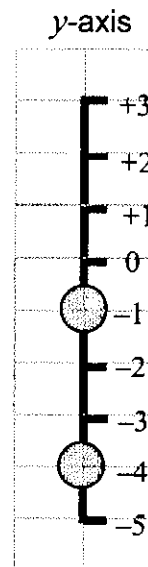
Grid paper can also be used to represent a number line to compare numbers vertically (up and down), similar to the  $y$ -axis of the coordinate plane.

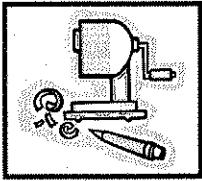
Select a location along the line to represent zero, and label the number line. Numbers become greater as you move up the number line.

### Example

Compare  $-1$  and  $-4$

$-1$  is further up the number line.  
Therefore,  $-1$  is greater than ( $>$ )  $-4$ .





## Practice: Ordering Integers

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1. Pedro, a golf caddy, kept track of one golfer's scores on nine holes of golf. The scores were as follows:  $+2$ ,  $0$ ,  $-4$ ,  $+1$ ,  $-2$ ,  $-3$ ,  $+3$ ,  $-1$ ,  $+4$ . Rewrite these scores in order from least to greatest.
2. Selma and four of her friends all live in different cities. One day, while in a chat room, they began to discuss weather. The temperature where Selma lives was  $28^{\circ}\text{C}$ . It was  $17^{\circ}\text{C}$  at Kelly's,  $-2^{\circ}\text{C}$  at Amy's,  $-3^{\circ}\text{C}$  at Cora's and  $5^{\circ}\text{C}$  where Chelsie lives. Use a horizontal or vertical number line and/or grid paper to record these temperatures in order from least to greatest.

3. Order the following integers.

a)

$-2$	$+8$	$0$
$-6$	$-5$	
$+5$	$+1$	$-9$

b)

$+3$	$+10$	$-1$
$-14$	$+20$	
$-16$	$-20$	$-7$

c)

$-26$	$+85$	$+50$
$-62$	$-82$	
$+54$	$-11$	$-73$

4. Make a true statement by adding  $<$  or  $>$  to the circles below.

a)  $+5$    $+2$

b)  $+3$    $+8$

c)  $-2$    $+5$

d)  $+8$    $-2$

e)  $-4$    $-7$

f)  $-4$    $-9$

5. Circle the integer that is greater in each pair.

a) +6, +9

b) +5, -2

c) -3, -6

d) -1, +7

e) +2, 0

f) 0, +4

6. Circle the integer that is smaller in each pair.

a) +9, -1

b) +1, -5

c) -7, -3

d) +3, +4

e) -2, 0

f) -3, +4

7. Which integers are greater than +2?

+5, +1, +9, -3, -2, 0 \_\_\_\_\_

8. Which integers are less than +3?

+1, +6, -1, -7, 0, +6 \_\_\_\_\_

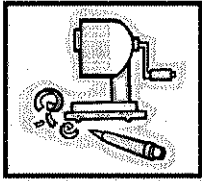
9. Which integers are greater than -3?

+4, 0, -6, -1, -9, +2 \_\_\_\_\_

10. Arrange the temperatures from the warmest to coldest.

-8°C, -23°C, +10°C, +21°C, -4°C \_\_\_\_\_

-18°C, -35°C, +12°C, +0°C, +3°C \_\_\_\_\_

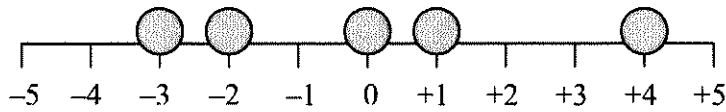


## Practice: Plotting Integers

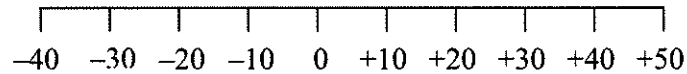
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1. Plot the integers on the number line. The first question is done for you as an example.

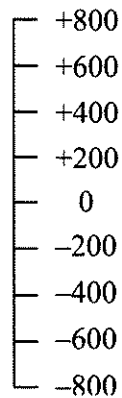
$-2, +4, 0, -3, +1$



- a)  $-35, +20, +5, +40, -15$



- b)  $-400, +300, +600, -100, -700$

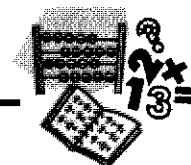


2. Draw and label a number line. Place the following integers on the number line.

$+10, -11, 0, -4, +7, -8$



## Adding Integers



The addition of integers can be done in a variety of ways, such as using number lines, manipulatives and a T-chart, calculators or shortcuts.

Parentheses (or brackets) are often used around integers and their positive/negative signs to make things less confusing when doing mathematical operations.

### Adding Integers Using a Number Line

Step 1: Draw a number line.

Step 2: Locate the first number on the number line.

Step 3: Move the number of spaces as shown by the second number.

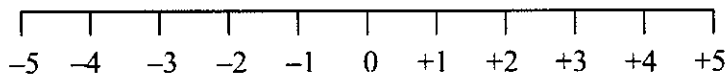
- Move to the **right** if the second integer is **positive**.
- Move to the **left** if the second integer is **negative**.

The number that you land on after moving the correct number of spaces to the left or right is the answer.

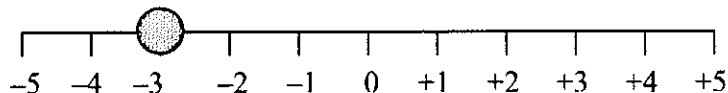
#### Examples

A)  $(-3) + (+5)$

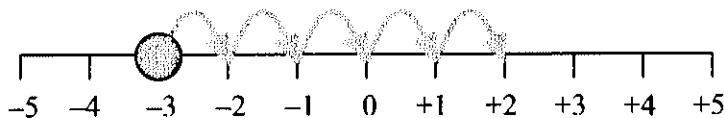
Step 1: Draw a number line.



Step 2: Locate the first number on the number line.

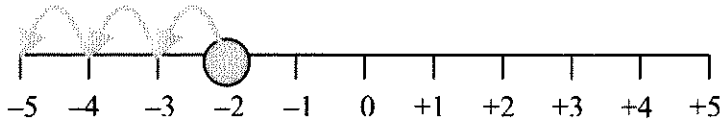


Step 3: From the first number, move five spaces to the right, because the second number is positive.



**Conclusion:**  $(-3) + (+5) = +2$

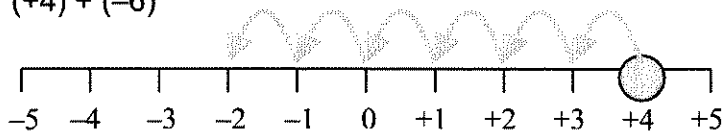
**B)**  $(-2) + (-3)$



$$(-2) + (-3) = -5$$

Move three places to the left of the first number, because the second number is negative.

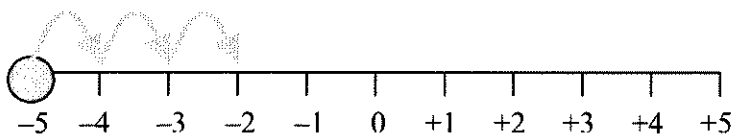
**C)**  $(+4) + (-6)$



$$(+4) + (-6) = -2$$

Move six places to the left of the first number, because the second number is negative.

**D)**  $(-5) + (+3)$



$$(-5) + (+3) = -2$$

Move three places to the right of the first number, because the second number is positive.



You can also use a vertical number line to help you think about adding integers.

Think of a hot air balloon floating through the points on a vertical number line.

**Positive numbers** are like puffs of air to lift the hot air balloon up.

**Negative numbers** are like sandbags to weigh the hot air balloon down.

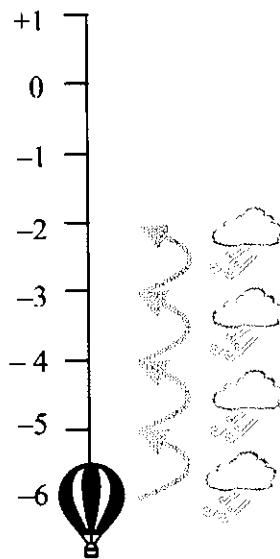
To add integers, place the first number on the number line. Look at what the second number is (positive = a puff of air; negative = a sandbag). Move along the number line the number of spaces as shown by the second number.

### Examples

A)  $(-6) + (+4) = \underline{\quad}$

You land here on  $(-2)$ .

Start the balloon here, at the first number.



The operation says to add a **positive** number  $(+4)$ . This means you have 4 puffs of air lifting the balloon up.

**Conclusion:**  $(-6) + (+4) = (-2)$

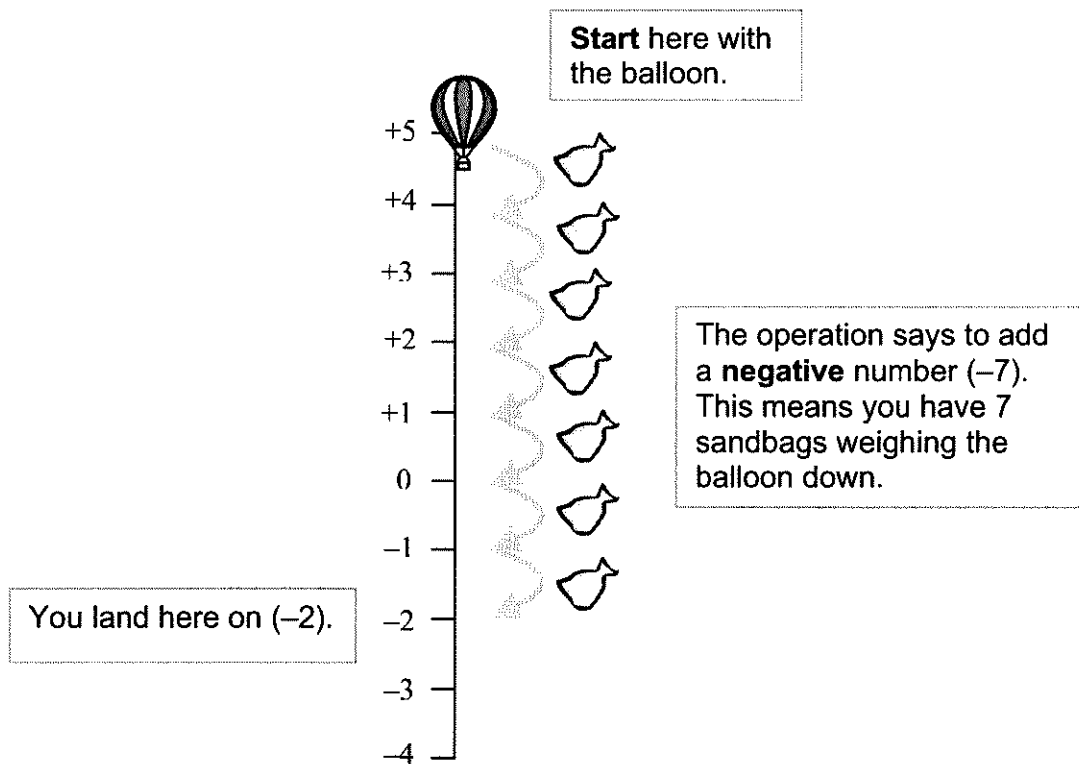
B)  $(+5) + (-7) = \underline{\quad}$

What does this mean?

Think of the second number as sandbags.

Start with  $(+5)$  and ADD  $(-7)$  or sandbags, which will bring down your hot air balloon.

Look on the vertical number line.



**Conclusion:**  $(+5) + (-7) = (-2)$

## Adding Integers Using Manipulatives and a T-Chart

**Manipulatives** are objects or shapes, such as coloured counters or chips, that are used to represent positive and negative numbers.



Manipulatives can be created using a computer. For example, copy a simple shape onto a word processing document.

Colour or shade the shape and make 10 copies. Each of these shapes will represent positive one (+1).

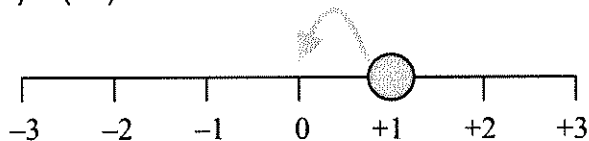
Create another shape and shade it a different colour. Make 10 copies. Each of these shapes will represent negative one (-1).

E.g.,     ●           ○  
              +1          -1

**One positive object and one negative object equal ZERO.**

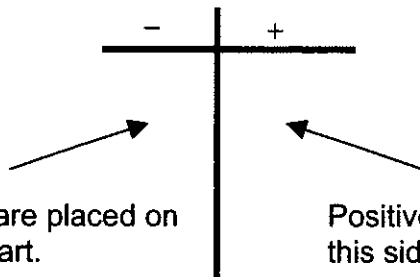
E.g., ● + ○ = 0

$(+1) + (-1) = 0$  can be shown on the number line below.



Placing manipulatives on a chart like the one below, or on opposite sides of a folded sheet of paper, helps in the addition of integers.

E.g.,

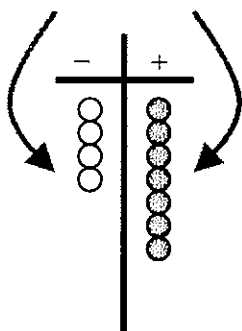


Negative objects are placed on this side of the chart.

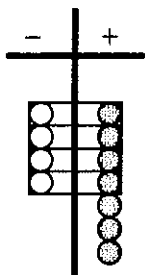
Positive objects are placed on this side of the chart.

**Examples**

A)  $(-4) + (+7)$

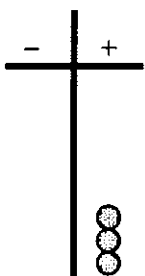


Step 1: Place objects on the chart to represent the value of each integer.



Step 2: Cross off or remove pairs of positive and negative objects. Remember:

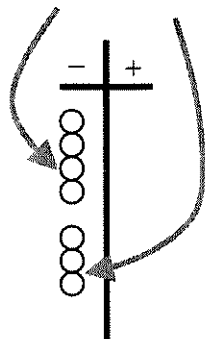
 = 0



The solution to the problem is the number of objects remaining. The sign is the same as the colour of the objects remaining and/or the sign of the column of the T-chart.

$(-4) + (+7) = +3$

- B) The temperature was  $-4^{\circ}\text{C}$ , then dropped another  $3^{\circ}\text{C}$ . What is the final temperature now?  $(-4) + (-3)$



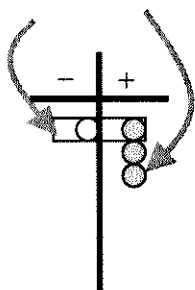
Add objects to the chart to represent the value of each integer.

There are no pairs of positive and negative integers, therefore the solution is the number of objects on the chart.

$$(-4) + (-3) = -7$$

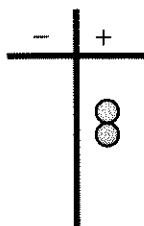
The temperature is now  $-7^{\circ}\text{C}$ .

- C) A golfer scored  $+3$  on one hole and then  $-1$  on the next hole. What is her score so far?  $(-1) + (+3)$



Add objects to the chart to represent the value of each integer.

Cross off or remove pairs of positive and negative objects.



The solution to the addition problem is the number of objects remaining on the chart, with the sign of the integer matching the colour of the counters/chips left.

$$(+3) + (-1) = +2$$

The golfer's score so far is  $+2$ .



## Calculator Method #2

### Example

$$(-3) + (+12)$$

1. If the first integer is negative, press the  $+/-$  key, then enter the value of the first integer.

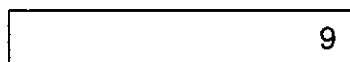


2. Press the  $+$  key.

3. If the second integer is negative, press the  $+/-$  key then enter the value of the second integer. In this example, the second integer is positive, so the  $+/-$  key is not pushed.



4. Press the  $=$  key to display the answer.



$$(-3) + (+12) = +9$$

### Example

Check out this example of adding integers using a calculator.

$$(-28) + (+12)$$

Method #1  $\boxed{2}$   $\boxed{8}$   $\boxed{+/-}$   $\boxed{+}$   $\boxed{1}$   $\boxed{2}$   $\boxed{=}$   $\boxed{-16}$

Method #2  $\boxed{+/-}$   $\boxed{2}$   $\boxed{8}$   $\boxed{+}$   $\boxed{1}$   $\boxed{2}$   $\boxed{=}$   $\boxed{-16}$

$$(-28) + (+12) = -16$$

## Adding Integers Using Shortcuts

Two shortcuts can be applied to help with the addition of integers.

### Adding integers with different signs

**Example**  $(+5) + (-2)$

Ignore the positive and negative signs for the moment and subtract the numbers.

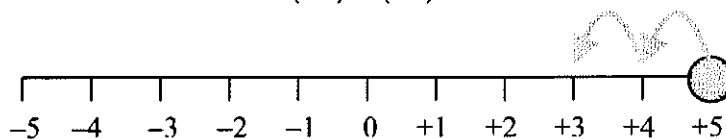
$$5 - 2 = 3$$

The sign of the answer is the sign of the largest number in the question.

$5 > 2$  The sign of the answer will be the sign in front of the 5 in the question.

$$(+5) + (-2) = +3$$

Proof:



### Adding integers with the same signs

**Example**  $(-1) + (-3)$

If both numbers are positive or negative, add the numbers.

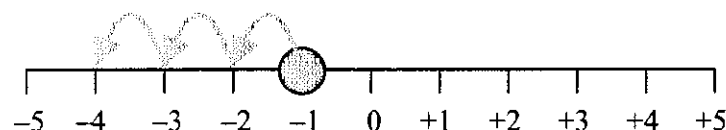
$$1 + 3 = 4$$

The sign of the numbers in the question becomes the sign of the answer.

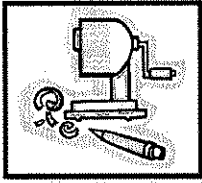
The signs of both numbers in the question were negative, so the sign of the answer will also be negative.

$$(-1) + (-3) = -4$$

Proof:







## Practice: Adding Integers

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1. What is the opposite integer? The first one is done for you.

a)

<u>Integer</u>	<u>Opposite</u>
+13	-13

b)

<u>Integer</u>	<u>Opposite</u>
-23	

c)

<u>Integer</u>	<u>Opposite</u>
+40	

d)

<u>Integer</u>	<u>Opposite</u>
-55	

2. Find the missing integer.

a)  $(+4) + ( \quad ) = +10$

b)  $(+6) + ( \quad ) = +4$

c)  $(-5) + ( \quad ) = -9$

d)  $(-7) + ( \quad ) = -5$

e)  $(+9) + ( \quad ) = +3$

f)  $(-4) + ( \quad ) = -11$

3. Use a number line or manipulatives to solve the following.

a)  $(+2) + (+6) + (-2) = \underline{\hspace{2cm}}$

b)  $(-3) + (+8) + (+2) = \underline{\hspace{2cm}}$

c)  $(-6) + (-1) + (-7) = \underline{\hspace{2cm}}$

d)  $(+9) + (-6) + (+4) = \underline{\hspace{2cm}}$

e)  $(+1) + (-7) + (+5) = \underline{\hspace{2cm}}$

f)  $(-7) + (+4) + (-12) = \underline{\hspace{2cm}}$

4. Write each problem in mathematical terms using integers. Choose a method to solve for the unknown, such as a number line, manipulatives or a calculator. The first one is done for you.

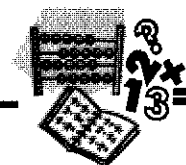
Imran has \$500.00 in his bank account. He purchased a personal CD player for \$295.00. How much money does Imran have left in his account?

$$(+\$500.00) + (-\$295.00) = \$205.00$$

**Imran has \$205.00 left in his account.**

- a) Chong woke up in the morning and checked the thermometer outside her window. It was  $-20^{\circ}\text{C}$ . Four hours later, the temperature dropped  $7^{\circ}\text{C}$ . What is the outside temperature now?
- b) David cuts and rakes grass in the neighbourhood. He earned \$40.00 on Monday, spent \$10.00 on Tuesday, spent \$5.00 on Wednesday, earned \$20.00 on Thursday, and earned \$10.00 on Friday. How much money did David have at the end of the week?
- c) A submarine starts at sea level and dives 50 m below the surface, then rises 20 m, and then finally dives 84 m. What is the submarine's final depth?
- d) Nina and her friends took up the sport of golf. They played their best on hole 8. Their scores for hole 8 were: +5, -1, 0, +4, -2, -1, +3. What was their combined score on the eighth hole?

## Subtracting Integers



The subtraction of integers can be demonstrated in a variety of ways, such as using number lines, manipulatives and a T-chart, calculator or shortcuts.

Parentheses are often used around integers and their positive/negative signs to reduce confusion when trying to perform mathematical operations.

### Subtracting Integers Using a Number Line

The process for subtracting integers is a lot like the process for adding them, except the direction is reversed.

Step 1: Draw a number line.

Step 2: Locate the first number on the number line.

Step 3: Move the number of spaces as shown by the second number.

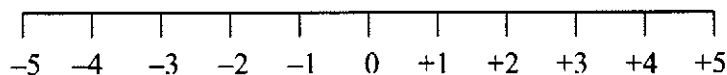
- Move to the **left** if the second integer is **positive**.
- Move to the **right** if the second integer is **negative**.

The number that you land on after moving the correct number of spaces to the left or right is the answer.

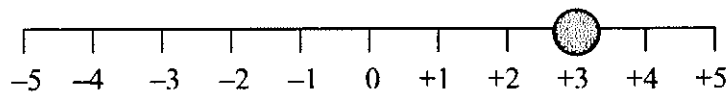
#### Examples

A)  $(+3) - (+5)$

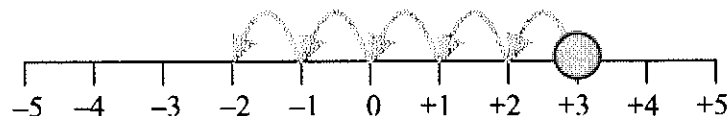
Step 1: Draw a number line using integers.



Step 2: Locate the first number on the number line.

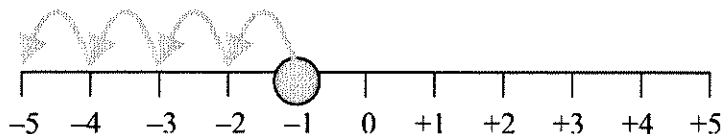


Step 3: From the first number, move five spaces to the left.



**Conclusion:**  
 $(+3) - (+5) = -2$

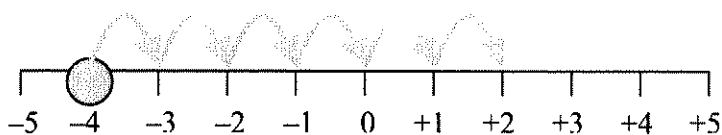
**B)  $(-1) - (+4)$**



$(-1) - (+4) = -5$

Move four places to the left of the first number because the second number is positive.

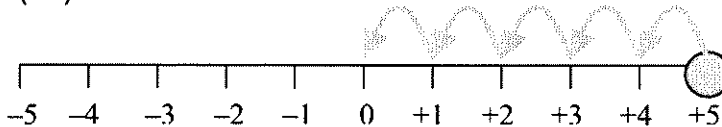
**C)  $(-4) - (-6)$**



$(-4) - (-6) = +2$

Move six places to the right of the first number because the second number is negative.

**D)  $(+5) - (+5)$**



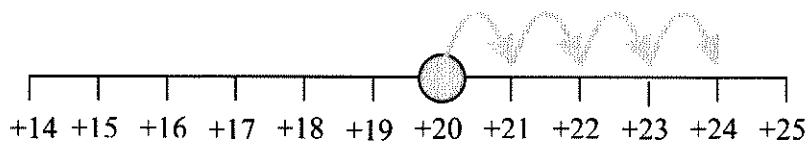
$(+5) - (+5) = 0$

Move five places to the left of the first number because the second number is positive.

- E) On Tuesday, the temperature at noon was  $+20^{\circ}\text{C}$ . The temperature increased by  $4^{\circ}\text{C}$  in the next two hours. What was the final temperature at 2:00 p.m.?**

Add the temperature and the increase.

$$\begin{aligned} (+20^{\circ}\text{C}) + (+4^{\circ}\text{C}) &= ? \\ 20^{\circ}\text{C} + 4^{\circ}\text{C} &= 24^{\circ}\text{C} \end{aligned}$$



The temperature at 2:00 p.m. was  $+24^{\circ}\text{C}$ .

You can also use a vertical number line to help you think about subtracting integers.

Think of a hot air balloon floating through the points on a vertical number line.

**Positive numbers** are like puffs of air to lift the hot air balloon up.

**Negative numbers** are like sandbags to weigh the hot air balloon down.

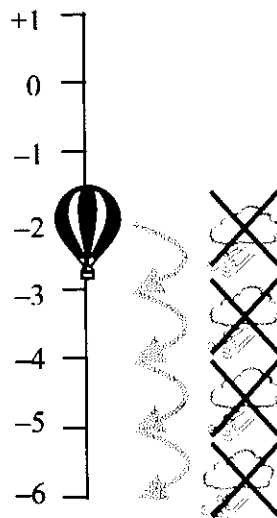
To subtract integers, place the first number on the number line. Look at what the second number is (positive = a puff of air; negative = a sandbag). Move along the number line the number of spaces as shown by the second number. Remember that when you're subtracting, you have to **remove** that number of puffs of air (which will cause the balloon to sink) or sandbags (which will cause the balloon to rise).

### Examples

A)  $(-2) - (+4) = \underline{\quad}$

Start the balloon here, at the first number.

You land here on  $(-6)$ .



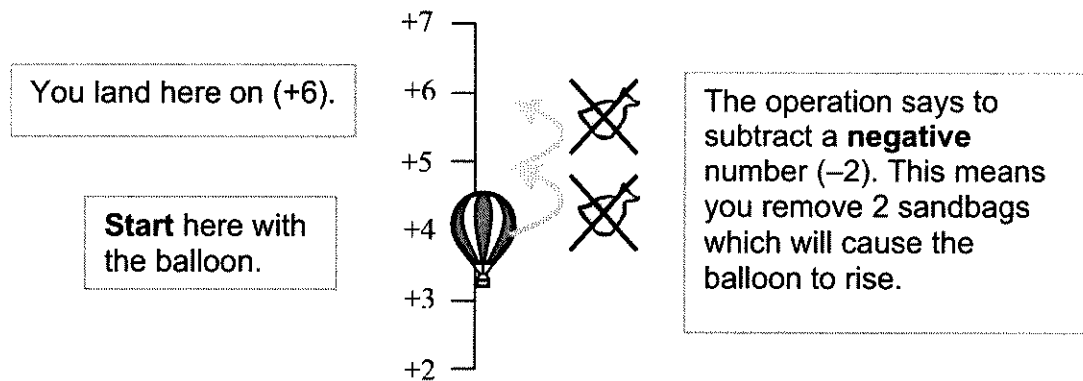
The operation says to subtract a **positive** number (+4). This means you remove 4 puffs of air, so the balloon will sink.

**Conclusion:**  $(-2) - (+4) = (-6)$

- B)  $(+4) - (-2) = \underline{\hspace{2cm}}$   
 What does this mean?

Think about the second number as puffs of air or sandbags. Start with  $(+4)$  and SUBTRACT  $(-2)$ . This means you remove 2 sandbags, which will lighten your hot air balloon and the balloon will rise.

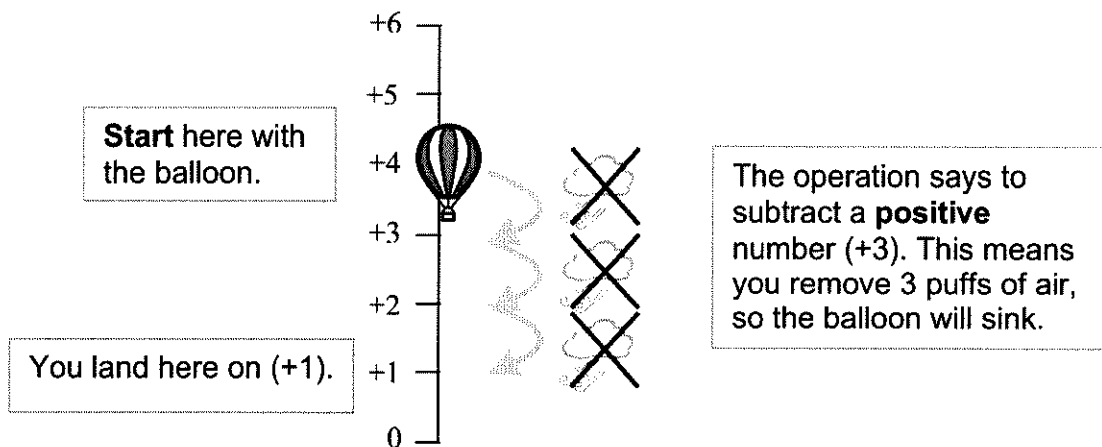
Look on the vertical number line.



**Conclusion:**  $(+4) - (-2) = (+6)$

- C)  $(+4) - (+3) = \underline{\hspace{2cm}}$

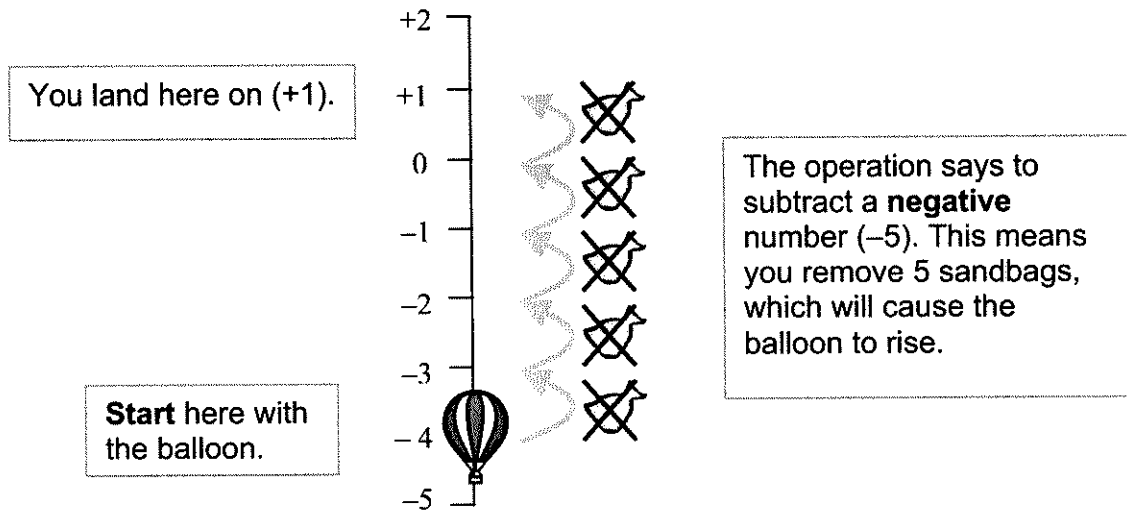
Remember, positive numbers are like puffs of air, so you have to subtract or remove 3 puffs of air. This will cause the balloon to sink.



**Conclusion:**  $(+4) - (+3) = (+1)$

D)  $(-4) - (-5) = \underline{\quad}$

Remember, negative numbers are like sandbags, so you have to subtract or remove 5 sandbags. This will lighten your hot air balloon and cause it to rise.

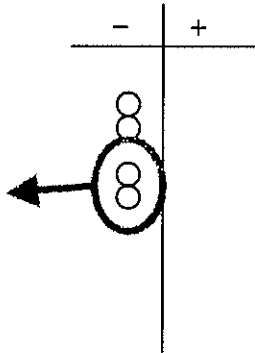


**Conclusion:**  $(+4) - (-5) = (+1)$

# Subtracting Integers Using Manipulatives and a T-chart

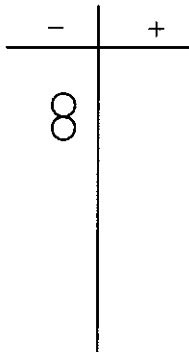
## Examples

A)  $(-4) - (-2)$



Step 1: Place objects on the chart to represent the value of the first integer.

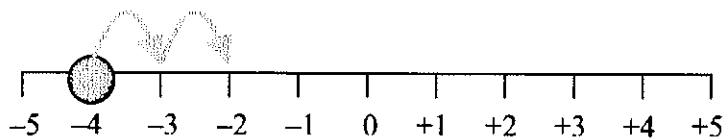
Step 2: Remove or subtract the value of the second integer from the chart.



The solution to the problem is the number of objects remaining. The colour of the objects and the sign on the T-chart is the sign used in the solution.

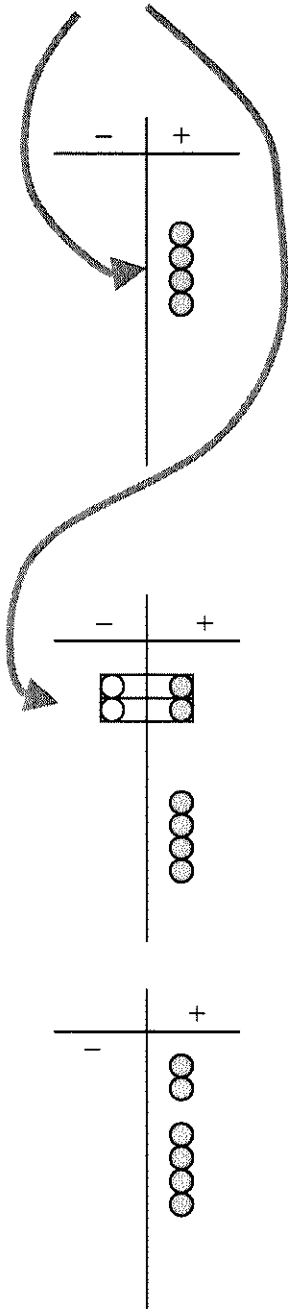
$$(-4) - (-2) = -2$$

Proof:





**B)  $(+4) - (-2)$**




Step 1: Place the value of the first integer onto the chart using the correctly coloured objects.

Step 2: Remove or subtract the value of the second integer  $(-2)$ . To solve the equation, we need to remove 2 negative objects, but there are no negative objects to remove or subtract from the chart.

Two negative objects CAN be brought onto the chart without changing the original question by bringing them in pairs with two positive objects.

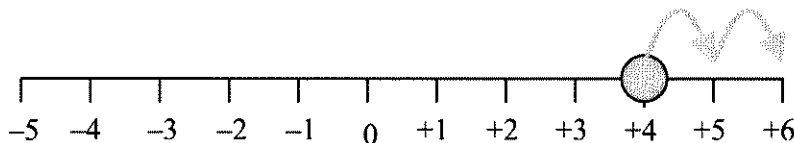
Remember: Each pair of objects equals zero, because:  $(-1) + (+1) = 0$

 = 0

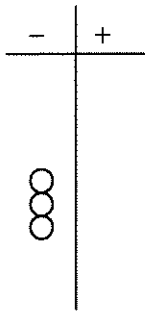
The solution to the question is the number of objects remaining.

$(+4) - (-2) = +6$

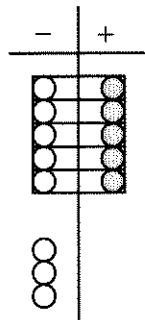
Proof:



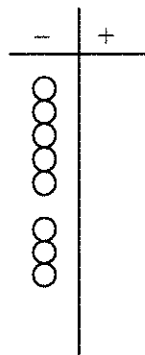
**C)  $(-3) - (+5)$**



Step 1: Add objects to the chart to represent the value of the first integer.



Step 2: We need to remove 5 positive objects. Five positive objects CAN be brought onto the chart without changing the original question by bringing them in pairs, with five negative objects.

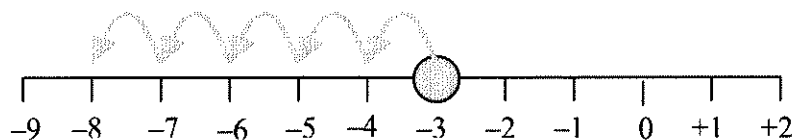


Step 3: Remove 5 positive objects.

The solution to the subtraction problem is the number of objects remaining on the chart.

$$(-3) - (+5) = -8$$

Proof:





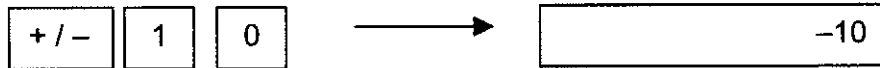


## Calculator Method #2

### Example

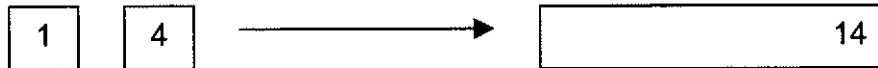
E.g.,  $(-10) - (+14)$

1. If the first integer is negative, press the  $+/-$  key on the calculator, then enter the value of the first integer.

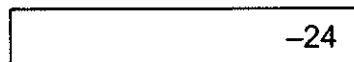


2. Press the  $-$  key.

3. If the second integer is negative, press the  $+/-$  key, then enter the value of the second integer. In this example, the second integer is positive so the  $+/-$  key is not pushed.



4. Press the  $=$  key to display the answer.

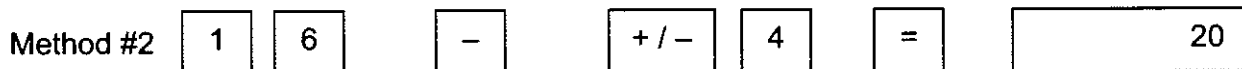
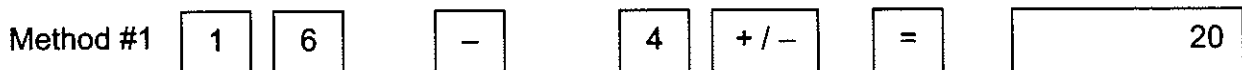


$$(-10) - (+14) = -24$$

### Example

Check out these examples of subtracting integers using a calculator.

**A)  $(+16) - (-4)$**



$$(+16) - (-4) = +20$$

**B)** On Monday, the temperature increased from  $+4^{\circ}\text{C}$  to  $+19^{\circ}\text{C}$ . By how many degrees did the temperature change during the day?

Subtract the lower temperature from the higher temperature.

$$19^{\circ}\text{C} - 4^{\circ}\text{C} = 15^{\circ}\text{C}$$

Use a calculator.

1	9	-	4	=	15
---	---	---	---	---	----

The change in temperature was  $+15^{\circ}\text{C}$ .

## Subtracting Integers Using Shortcuts

Subtraction of integers can be thought of as “adding the opposite.” That is, changing the subtraction question into an addition question.

### Example

$$(+5) - (-2)$$

- The subtraction sign ( $-$ ) is changed to an addition sign ( $+$ ) between the two integers.

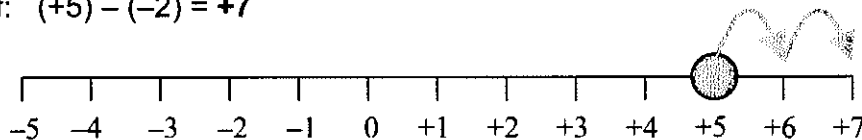
$$(+5) + (-2)$$

- The sign of the second integer is also changed to its opposite sign. In the example, the opposite of  $-2$  is  $+2$ .

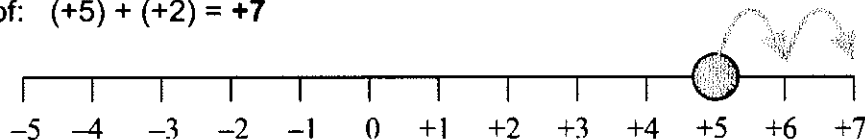
$$(+5) + (+2)$$

Apply the rules for adding integers.

Proof:  $(+5) - (-2) = +7$



Proof:  $(+5) + (+2) = +7$





Weather forecasters or meteorologists must be able to add and subtract integers to compare temperatures from one region to another and determine temperature increases or decreases during the day.

Meteorologists create number sentences or mathematical questions and then solve these questions to determine temperature changes or differences.

### Example

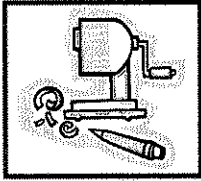
At 6:00 a.m., the temperature was  $-18^{\circ}\text{C}$ . Within the next hour, the temperature dropped another  $5^{\circ}\text{C}$ . It then rose by  $3^{\circ}\text{C}$ , only to drop again  $6^{\circ}\text{C}$ . What was the final temperature?

$(-18^{\circ}\text{C}) + (-5^{\circ}\text{C}) + (+3^{\circ}\text{C}) + (-6^{\circ}\text{C})$ . The final temperature is  $-26^{\circ}\text{C}$ .

What is the difference between the initial and final temperatures?

$$(-18^{\circ}\text{C}) - (-26^{\circ}\text{C}) = +8^{\circ}\text{C}$$





## Practice: Subtracting Integers

---

1. Answer the following by changing the subtraction equation into an addition equation. The first two have been done for you.

$$\begin{aligned} \text{a) } & (+2) - (-7) \\ & = (+2) + (+7) \\ & = +9 \end{aligned}$$

$$\begin{aligned} \text{b) } & (-2) - (+6) \\ & = (-2) + (-6) \\ & = -8 \end{aligned}$$

$$\begin{aligned} \text{c) } & (+7) - (+4) \\ & = (+7) + (- \quad ) \\ & = \end{aligned}$$

$$\begin{aligned} \text{d) } & (-8) - (-4) \\ & = (-8) + ( \quad ) \\ & = \end{aligned}$$

$$\begin{aligned} \text{e) } & (-2) - (-5) \\ & = \\ & = \end{aligned}$$

$$\begin{aligned} \text{f) } & (+5) - (-7) \\ & = \\ & = \end{aligned}$$

2. Write each problem as a mathematical sentence or question using integers. Then choose a calculation method, such as a number line, manipulatives or a calculator, to find the solution to each problem.

- a) The temperature increases from  $-15^{\circ}\text{C}$  to  $+2^{\circ}\text{C}$  during the day. What is the difference between these two temperatures?
- b) The temperature drops from  $-13^{\circ}\text{C}$  to  $-24^{\circ}\text{C}$ . What is the difference between these two temperatures?
- c) A climber starts at an elevation of  $-180$  m and climbs to an elevation of  $240$  m. How high did this person climb in total?
- d) The temperature of a cup of hot chocolate is  $50^{\circ}\text{C}$ . A frozen treat has a temperature of  $-10^{\circ}\text{C}$ . What is the temperature difference between these two items?

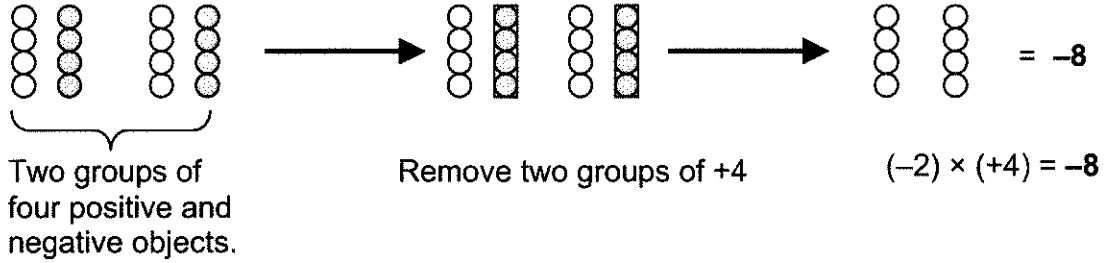




C)  $(-2) \times (+4)$   $\longrightarrow$

Remove two groups of  $\swarrow$   $\nwarrow$  four positive objects

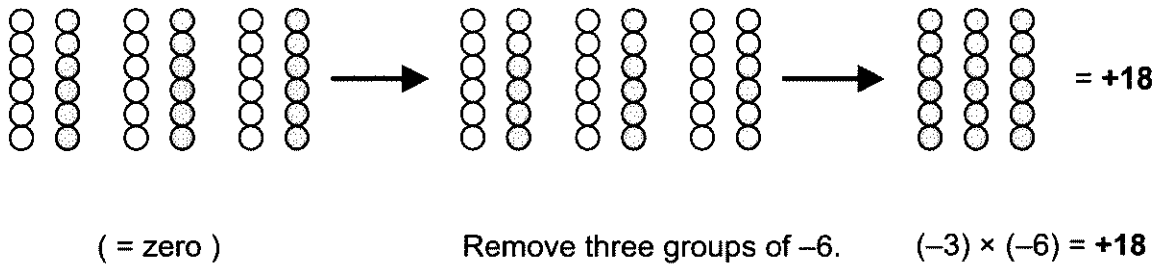
There are not any positive objects to remove, so **groups of zeros** must be added until two groups of four positive objects can be removed.



D)  $(-3) \times (-6)$   $\longrightarrow$

Remove three groups of  $\swarrow$   $\nwarrow$  six negative objects

There are not six negative objects to remove, so **groups of zeros** must be added until three groups of six negative objects can be removed.



## Multiplying Integers Using Pencil and Paper, and Sign Rules

The following sign rules can be used to find solutions when multiplying integers.

Step 1: Ignore the signs of the integers and multiply two numbers together to find the product.

Step 2: Apply the sign rules below to determine the sign of the product.

- If the signs are both the same, the product will be positive.

$$\begin{aligned} (+) \times (+) &= + \\ (-) \times (-) &= + \end{aligned}$$

- If the signs are different, the product will be negative.

$$\begin{aligned} (+) \times (-) &= - \\ (-) \times (+) &= - \end{aligned}$$

### Examples

A)  $(-2) \times (-12)$   
 $2 \times 12 = 24$   $\longrightarrow$   $= +24$   $(+4) \times (+5) = +20$   
 $(-) \times (-) = (+)$

B)  $(+4) \times (+5)$   
 $4 \times 5 = 20$   $\longrightarrow$   $= +20$   $(+4) \times (+5) = +20$   
 $(+) \times (+) = (+)$

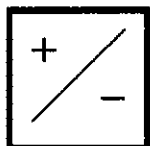
C)  $(-2) \times (+9)$   
 $2 \times 9 = 18$   $\longrightarrow$   $= -18$   $(-2) \times (+9) = -18$   
 $(-) \times (+) = (-)$

D)  $(-4) \times (-7)$   
 $4 \times 7 = 28$   $\longrightarrow$   $= +28$   $(-4) \times (-7) = +28$   
 $(-) \times (-) = (+)$



## Multiplying Integers Using a Calculator

Calculators can be used to multiply integers if the calculator has an integer button. On many models, the integer button looks like this:



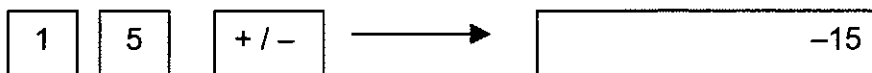
Depending on the model of calculator, the multiplication of integers can be done using one of two methods.

### Calculator Method #1

**Example**

$$(-15) \times (+3)$$

1. Enter the value of the first integer, followed by the  $\boxed{+/-}$  key only if the integer is negative.

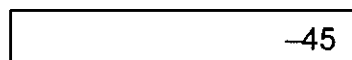


2. Press the  $\boxed{\times}$  key.

3. Enter the value of the second integer, followed by the  $\boxed{+/-}$  key only if the integer is negative. In this example, the second integer is positive so the  $\boxed{+/-}$  key is not pushed.



4. Press the  $\boxed{=}$  key to display the answer.



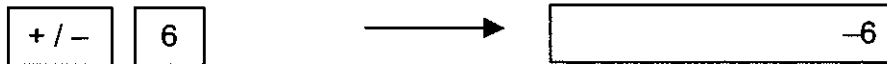
$$(-15) \times (+3) = -45$$

## Calculator Method #2

### Example

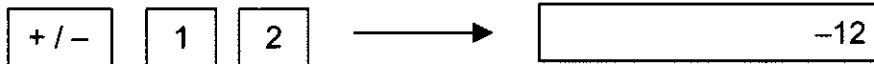
$(-6) \times (-12)$

1. If the first integer is negative, press the  $\boxed{+/-}$  key on the calculator, then enter the value of the first integer.



2. Press the  $\boxed{\times}$  key.

3. If the second integer is negative, press the  $\boxed{+/-}$  key on the calculator, then enter the value of the second integer.



4. Press the  $\boxed{=}$  key to display the answer.



### Example

$(+20) \times (-4)$

Method #1



Method #2



$(+20) \times (-4) = -80$

## Solving Word Problems

Check out these examples of writing mathematical statements from word problems.

### Examples

- A) A store loses fifty dollars a day over six days. How much money was lost in total over these six days?

$$(-50) \times (+6) = ?$$

(Using a calculator)

+ / -	5	0	x	6	=	-300
-------	---	---	---	---	---	------

A total of \$300.00 was lost by the end of six days.

A negative value of money represents a LOSS of money.

- B) The temperature rises  $+3^{\circ}\text{C}$  each hour for four hours. By how many degrees Celsius did the temperature rise in four hours?

$$(+3) \times (+4) = ?$$

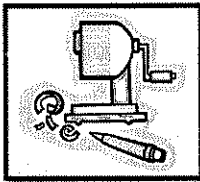
(Using sign rules)

$$\begin{aligned} 3 \times 4 &= 12 \\ (+) \times (+) &= (+) \end{aligned}$$

$$(+3) \times (+4) = +12$$

The temperature increased by a total of  $+12^{\circ}\text{C}$ .





## Practice: Multiplying Integers

---

1. Solve the following by selecting an appropriate strategy, such as manipulatives, calculator or pencil and paper.

a)  $(-2) \times (-8) =$

b)  $(+4) \times (-3) =$

c)  $(+5) \times (+12) =$

d)  $(-5) \times (-10) =$

e)  $(-9) \times (-7) =$

f)  $(+11) \times (+20) =$

g)  $(+10) \times (-5) =$

h)  $(+6) \times (-7) =$

i)  $(-15) \times (+2) =$

j)  $(+12) \times (-12) =$

2. Solve the following by selecting an appropriate strategy, such as manipulatives, calculator or pencil and paper.

a)  $(+40) \times (+60) =$

b)  $(-20) \times (-40) =$

c)  $(-35) \times (-5) =$

d)  $(+20) \times (-15) =$

e)  $(+35) \times (-12) =$

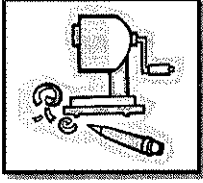
f)  $(-42) \times (+17) =$

g)  $(-33) \times (-100) =$

h)  $(+25) \times (+115) =$

i)  $(+54) \times (-152) =$

j)  $(-123) \times (+54) =$



## Practice: Solving Word Problems

---

Write each problem as a mathematical statement using integers. Solve using a variety of strategies, such as manipulatives, pencil and paper or calculator. The first one is done for you.

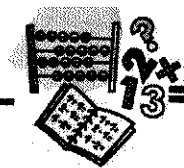
1. The temperature falls  $3^{\circ}\text{C}$  each hour for eight hours. What is the total change in temperature over this eight-hour period?

$$(-3^{\circ}\text{C}) \times 8 = -24^{\circ}\text{C}$$

The total change in temperature over the eight-hour period is  $-24^{\circ}\text{C}$ .

2. A submarine dives 20 m each minute for 15 minutes. What is the total change in depth after 15 minutes?
3. A stock loses \$1.50 each day over a five-day period. What is the total change in the value of the stock at the end of five days?
4. The temperature rises  $2^{\circ}\text{C}$  each hour from 6:00 a.m. to 3:00 p.m. What is the total change in temperature during this period of time?
5. Abdul earns \$13.00 in interest each month for six months. How much interest will Abdul have earned at the end of six months?

## Dividing Integers



Division of integers can be demonstrated in a variety of ways such as using pencil and paper and sign rules, or a calculator.

### Dividing Integers Using Pencil and Paper, and Sign Rules

The following sign rules can be used to find solutions when dividing integers.

Step 1: Ignore the signs of the integers and divide the numbers to find the quotient.

Step 2: Apply the sign rules below to determine the sign of the quotient.

- If the signs are both the same, the quotient will be positive.

$$\begin{aligned} (+) \div (+) &= + \\ (-) \div (-) &= + \end{aligned}$$

- If the signs are different, the quotient will be negative.

$$\begin{aligned} (+) \div (-) &= - \\ (-) \div (+) &= - \end{aligned}$$

### Examples

Check out these examples of division of integers using sign rules.

A)  $(-22) \div (-2)$

$$(-) \div (-) = (+)$$

↑ ↑  
Signs are the same



$$= +11$$

$$(-22) \div (-2) = +11$$

B)  $(+40) \div (-5)$

$$(+) \div (-) = (-)$$

↑ ↑  
Signs are different



$$= -8$$

$$(40) \div (-5) = -8$$

C)  $(-27) \div (+9)$



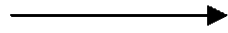
$= -3$

$(-27) \div (+9) = -3$

$(-) \div (+) = (-)$

Signs are different

D)  $(-49) \div (-7)$



$= +7$

$(-49) \div (-7) = +7$

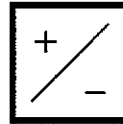
$(-) \div (-) = (+)$

Signs are the same

## Dividing Integers Using a Calculator



Calculators can be used to divide integers if the calculator has an integer button. On many models, the integer button looks like this:



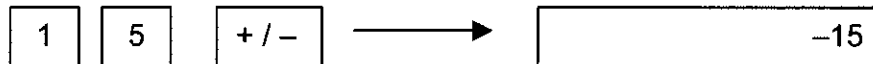
Depending on the model of calculator, the division of integers can be done using one of two methods.

### Calculator Method #1

**Example**

$(-15) \div (+3)$

1. Enter the value of the first integer, followed by the  $+/-$  key only if the integer is negative.



2. Press the  $\div$  key.

3. Enter the value of the second integer, followed by the  $+/-$  key only if the integer is negative. In this example, the second integer is positive so the  $+/-$  key is not pushed.



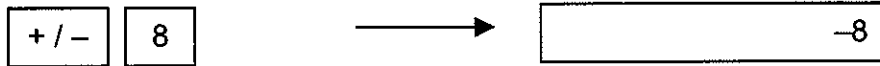
4. Press the  $=$  key to display the answer.



## Calculator Method #2

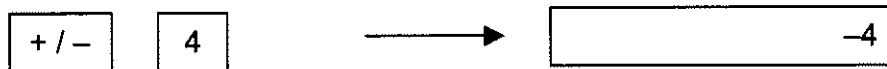
**Example**  $(-8) \div (-4)$

1. If the first integer is negative, press the  $+/-$  key on the calculator, then enter the value of the first integer.

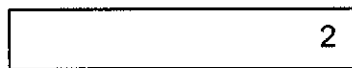


2. Press the  $\div$  key.

3. If the second integer is negative, press the  $+/-$  key on the calculator, then enter the value of the second integer.

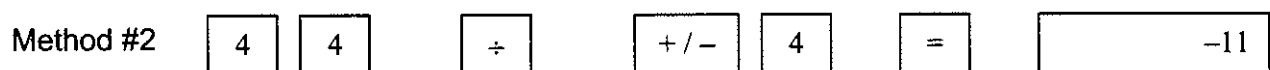


4. Press the  $=$  key to display the answer.



$(-8) \div (-4) = +2$

**Example**  $(+44) \div (-4)$



$(+44) \div (-4) = -11$

## Solving Word Problems

### Examples

Check out the examples of writing mathematical statements from word problems.

**A)** The value of a stock changes from +1.75 to -2.50 over a five-day period.

What is the average change in the value of the stock?

To calculate an average, the change between the values is calculated and divided by the number of days that the stock value changes over.

$$(-2.50) - (+1.75) = -4.25$$

$$(-4.25) \div 5 = -0.85$$

The average change in the value of the stock each day is **-0.85**.

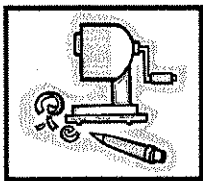
**B)** The temperatures over the past four days were  $-3^{\circ}\text{C}$ ,  $-5^{\circ}\text{C}$ ,  $+4^{\circ}\text{C}$  and  $+2^{\circ}\text{C}$ . What was the average temperature for the past four days?

Find the sum of the temperatures and divide by the number of days.

$$(-3) + (-5) + (+3) + (+1) = -4$$

$$(-4) \div 4 = -1$$

The average temperature over the past four days was  **$-1^{\circ}\text{C}$** .



## Practice: Dividing Integers

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1. Choose a strategy and solve the following.

a)  $(-24) \div (-8) =$

b)  $(+45) \div (-9) =$

c)  $(+40) \div (+4) =$

d)  $(-55) \div (+5) =$

e)  $(-10) \div (+2) =$

f)  $(-32) \div (+8) =$

g)  $(-72) \div (+9) =$

h)  $(+60) \div (-10) =$

i)  $(-40) \div (+5) =$

j)  $(-100) \div (+20) =$

2. Choose a strategy and solve the following.

a)  $(+120) \div (-20) =$

b)  $(-200) \div (-40) =$

c)  $(-350) \div (-5) =$

d)  $(+120) \div (-30) =$

e)  $(-450) \div (+25) =$

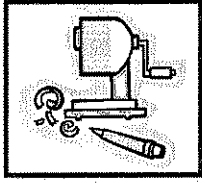
f)  $(+125) \div (+25) =$

g)  $(-1000) \div (-20) =$

h)  $(-3000) \div (+30) =$

i)  $(-2500) \div (+10) =$

j)  $(+1050) \div (-25) =$



## Practice: Solving Word Problems

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Write each problem as a mathematical statement. Solve using a variety of strategies, such as manipulatives, pencil and paper or a calculator. The first one is done for you.

1. The temperature is  $0^{\circ}\text{C}$  and falls  $4^{\circ}\text{C}$  each day. In how many days will the temperature be  $-20^{\circ}\text{C}$ ?

$$(-20^{\circ}\text{C}) \div (-4) = 5 \text{ days}$$

The temperature will be  $-20^{\circ}\text{C}$  in five days.

2. A deep sea diver sinks 2 m per second. How many seconds will it take the diver to reach a depth of 32 m?
3. Joey will pay \$24.00 each month for his entertainment system. How many months will it take to pay for his system if it cost \$432.00?
4. Janice bought a new mountain bike for \$990.00. If she pays \$45.00 a month, how many months will it take to pay for it?