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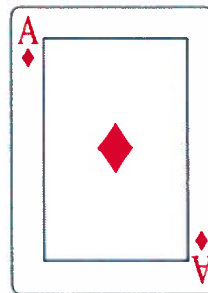
Start date: _____

Handin date: _____

MATH 9

Module 9

Data Analysis & Probability



Data Analysis and Probability

GETTING STARTED

Warm Up

9.1 Reading Scatter Plots

9.2 Drawing Scatter Plots

9.3 Lines of Best Fit

9.4 The Probability Formula

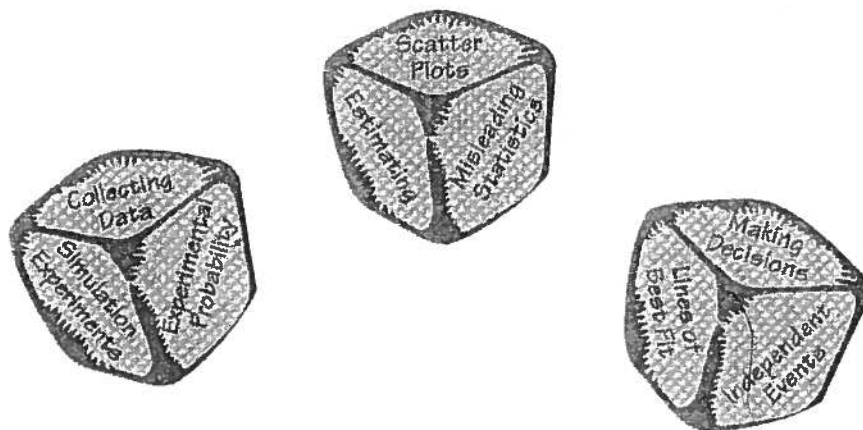
9.5 Independent Events

Review

Chapter Check

Problem Solving: Using the Strategies

Answers CHAPTER 9 Data Analysis and Probability



9.1 Reading Scatter Plots

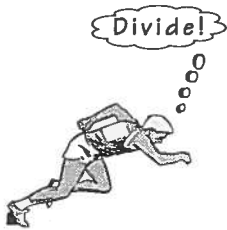
Problems and Applications

1. The scatter plot shows the winning times for 5 races at the 1992 Olympics.

- a) What is the approximate winning time for the 400-m race?

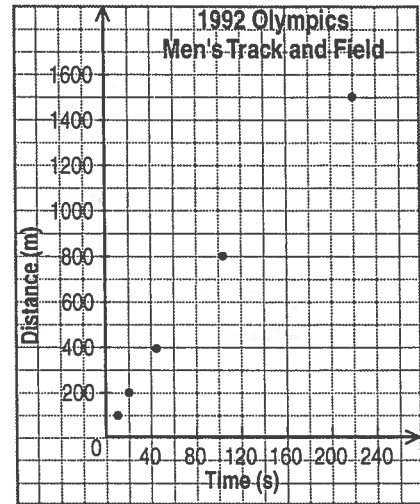
Sentence: _____

- b) About how many times greater was the winning time for the 800-m race than for the 100-m race?



$$\frac{\text{Winning time for 800-m race}}{\text{Winning time for 100-m race}} = \frac{\boxed{}}{\boxed{}}$$

$$= \underline{\hspace{2cm}}$$



Sentence: _____

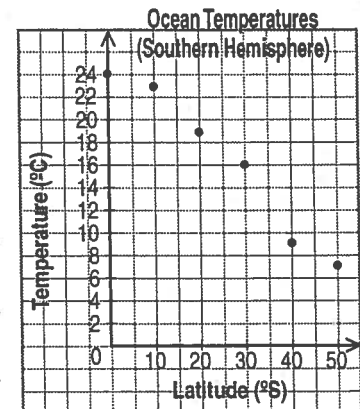
- c) If there had been a 1000-m race, what winning time would you estimate for it?

- d) Estimate how far a top male athlete can run in 3 min.

Hint: 3 min = _____ sec

2. The scatter plot shows how the average temperature of the ocean changes with latitude in the southern hemisphere.

- a) Describe the relationship between average temperature and latitude.



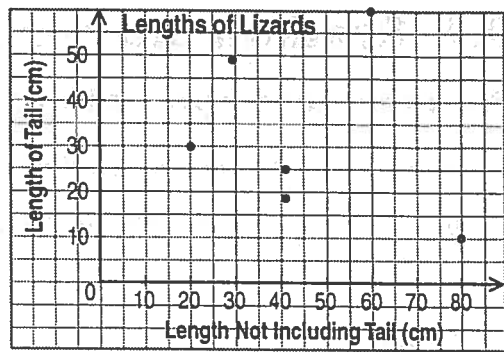
- b) Estimate the average ocean temperature at a latitude of

- i) 35° S _____ ii) 22° S _____

- c) At what latitude is the border between the United States and Alberta?



3. The scatter plot shows the lengths of lizards, not including their tails, and the lengths of their tails.



a) Estimate the length of the tail of each of the following.

i) a lizard 20 cm in length _____

ii) a lizard 60 cm in length _____

iii) a lizard 80 cm in length _____

b) Does the scatter plot suggest a relationship between the lengths of the lizards and the lengths of their tails? _____

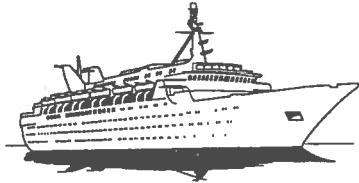
Explain. _____

c) Do all scatter plots show a relationship? _____

9.2 Drawing Scatter Plots

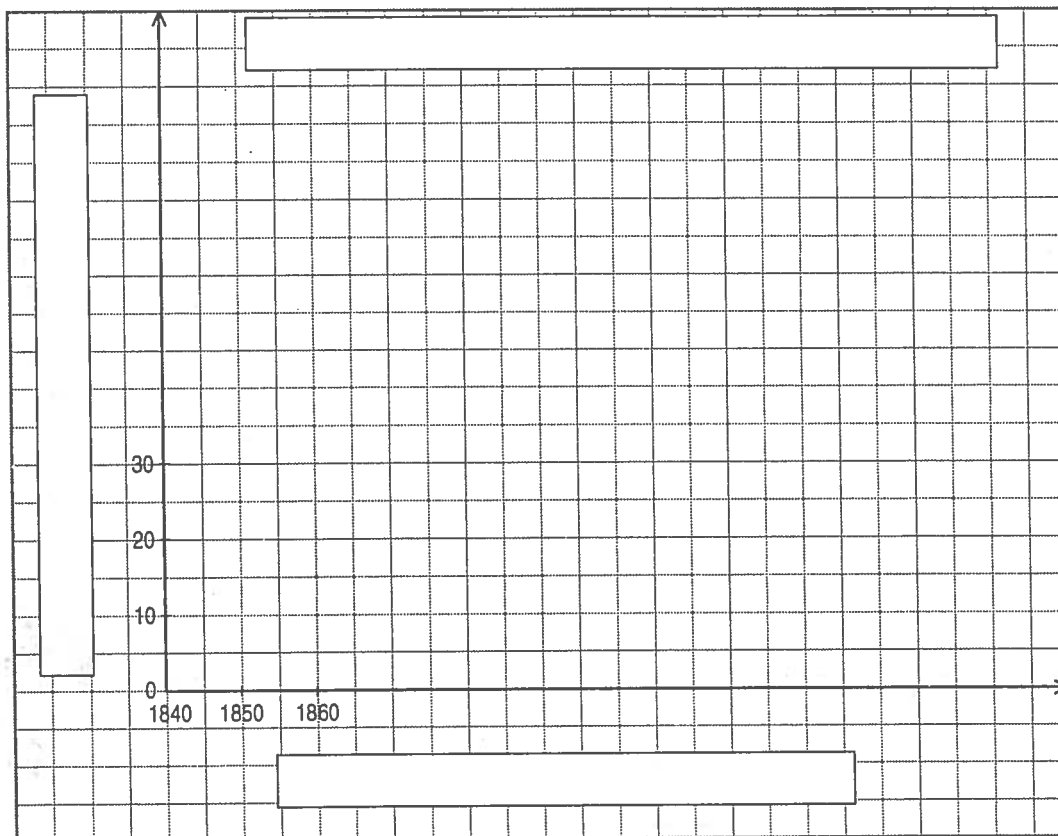
Problems and Applications

1. The table shows the year and the speeds of the winners for the fastest crossing of the Atlantic Ocean by a liner.



Year	Winner	Speed (km/h)
1840	<i>Britannia</i>	19.6
1863	<i>Scotia</i>	25.9
1882	<i>Alaska</i>	31.9
1897	<i>Kaiser Wilhelm der Grosse</i>	41.5
1909	<i>Mauretania</i>	48.0
1929	<i>Bremen</i>	51.7
1938	<i>Queen Mary</i>	58.7
1952	<i>United States</i>	66.0

- a) Draw a scatter plot of speed versus year.



1. Label each axis.
2. Plot the points.
3. Give a title.

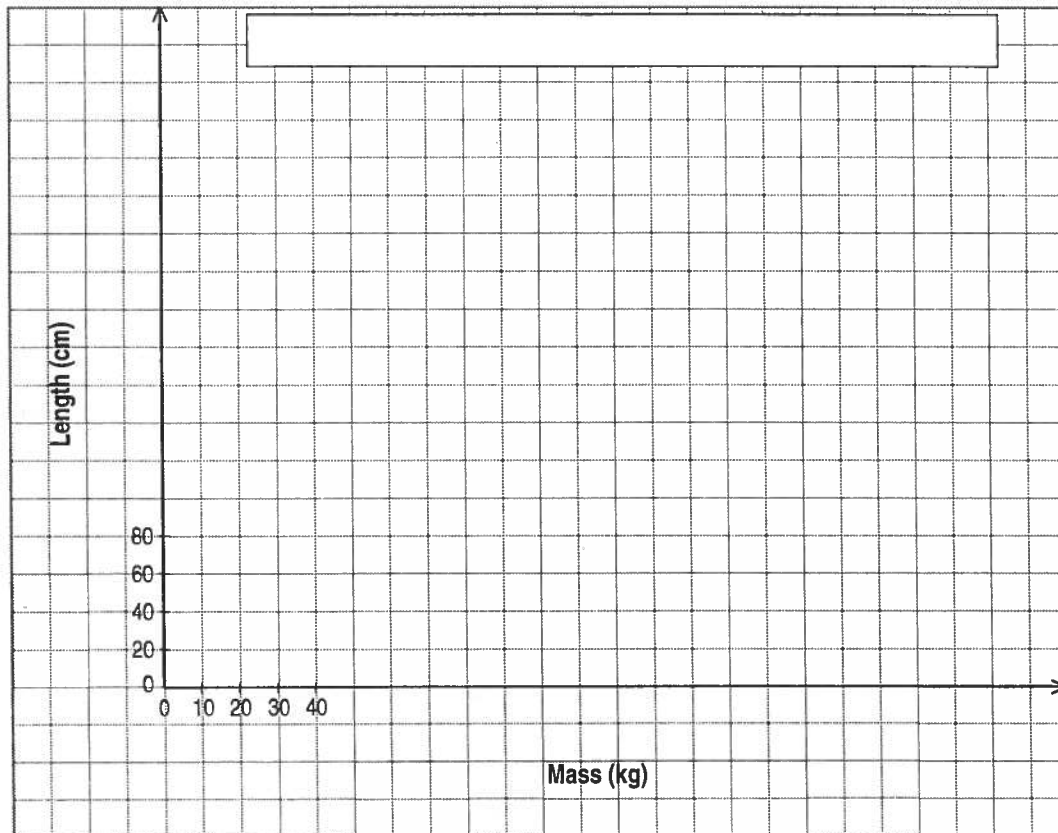
- b) Describe any relationship you see. _____



2. The table shows the *masses* of different types of cats, and the *lengths*, from the nose to the end of the tail.

Type of Cat	Mass (kg)	Length (cm)
Lion	180	300
Lioness	140	270
Cheetah	45	180
Mountain Lion	90	240
Jaguar	140	260
Leopard	70	265
Tiger	190	270
Tigress	135	240
Lynx	30	90

a) Display the data in a scatter plot of **mass** versus **length**.



1. Label each axis.
2. Plot the points.
3. Give a title.



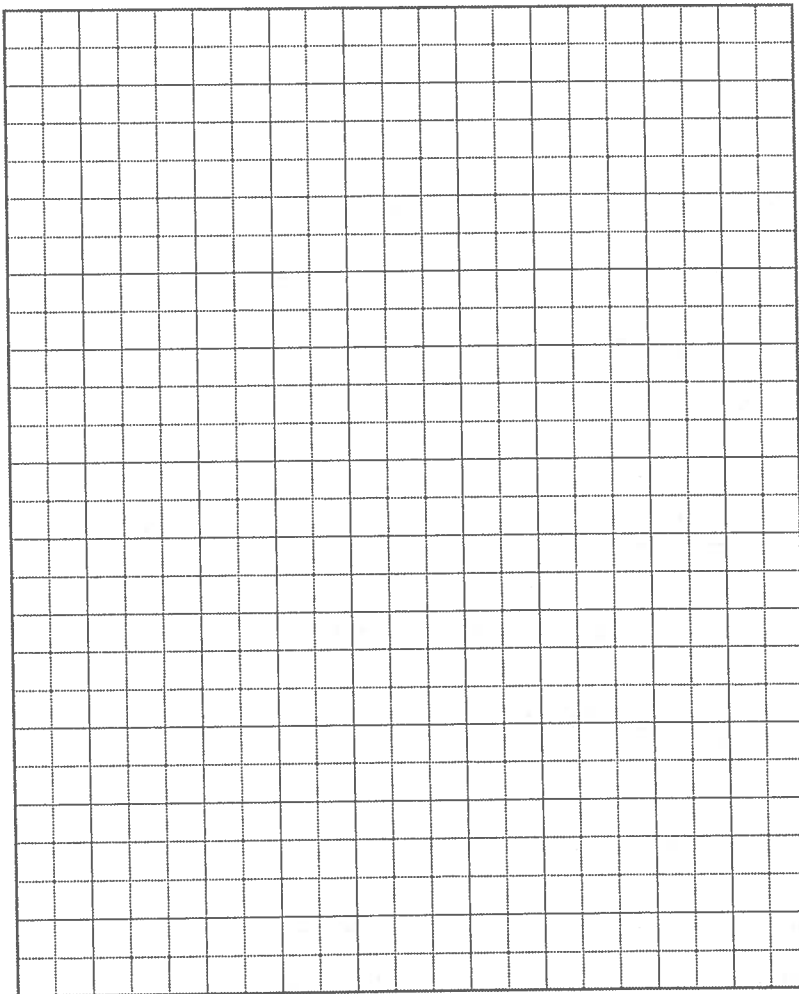
b) Describe any relationship you see. _____



3. The table shows the percent of Canadians under 20 years of age in different years.

Year	Percent of Canadians Under 20
1966	42
1971	39
1976	36
1981	32
1986	29
1991	27

a) Display the data on a scatter plot of percent versus year.



1. Draw and label each axis.
2. Plot the points.
3. Give the graph a title.



b) Describe any relationship you see. _____

4. Students travel to school in different ways — by car, on foot, and so on.

a) Complete the chart.

Ask 10 students who travel to school in the same way as you do, to estimate

- the distance, in kilometres, from their home to school

and

- the average time, in minutes, the trip takes.

Student's Name	Distance (km) From School	Average Time (min) the Trip Takes

b) Use your data to draw a scatter plot of **distance** versus **time**.

Use graph paper.

c) Describe any relationship you see. _____

d) List four factors that can affect the length of time for the trip.

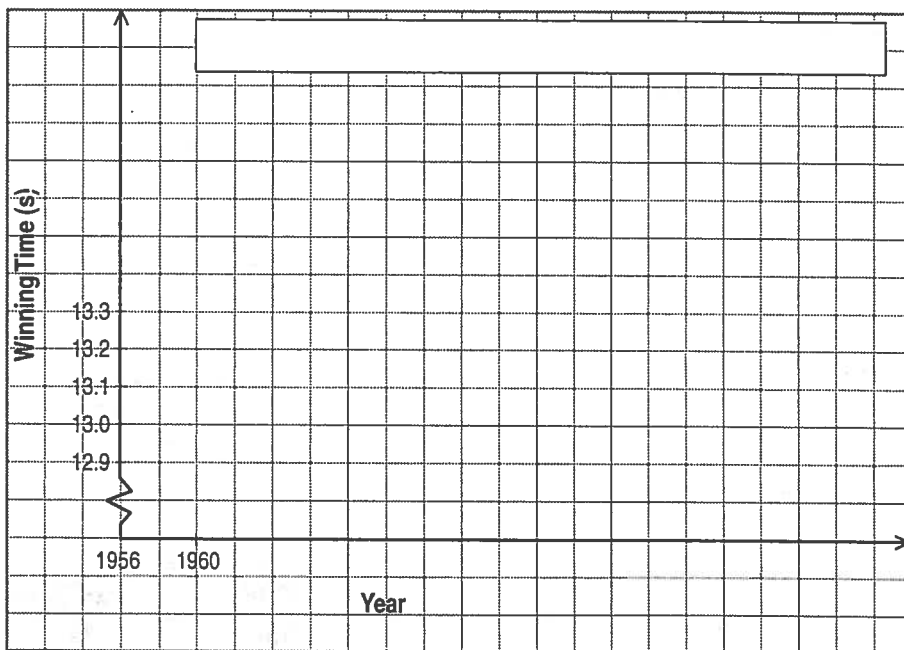
9.3 Lines of Best Fit

Problems and Applications

1. The table shows some winning times in the men's 110-m hurdles at the Olympic Games.

Year	Winner	Winning Time (s)
1956	Calhoun (U.S.)	13.5
1960	Calhoun (U.S.)	13.8
1964	Jones (U.S.)	13.6
1968	Davenport (U.S.)	13.3
1972	Milburn (U.S.)	13.24
1976	Drut (France)	13.30
1980	Munkelt (East Germany)	13.39
1984	Kingdom (U.S.)	13.20
1988	Kingdom (U.S.)	12.98
1992	McKoy (Canada)	13.12

- a) Draw a scatter plot of winning time versus year.



1. Label each axis.
2. Plot the points.
3. Give the graph a title.

- b) Draw a line of best fit.

Line of Best Fit

- a **straight line** on a graph that lies as close as possible to all the points on the graph
- There are about as many **points above** the line as there are **below** the line.

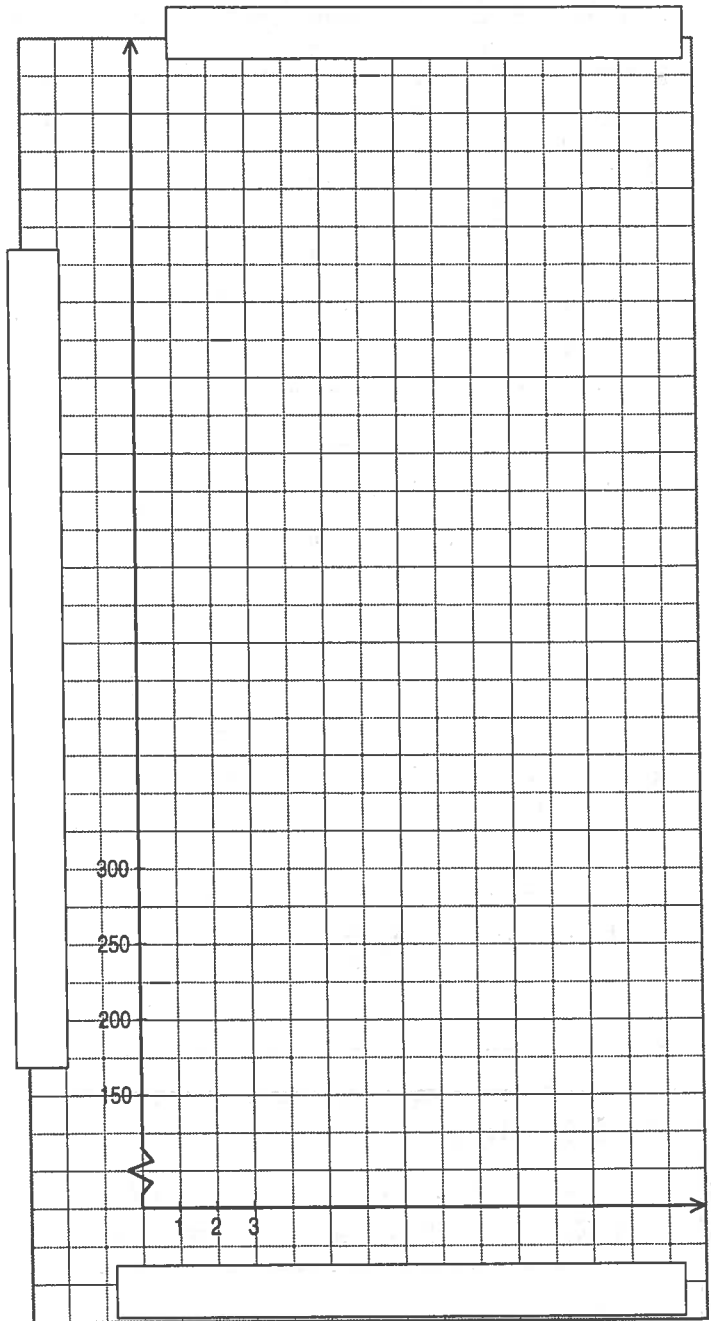
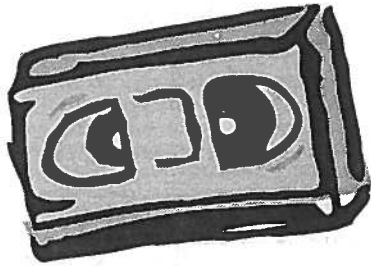
To extrapolate, make the line of best fit longer.

- c) Extrapolate to estimate the winning time in 1952.

- d) Estimate the winning time in 2000.

2. The table shows the number of rentals of a video in each week for the first ten weeks.

Week	Number of Rentals
1	642
2	635
3	762
4	695
5	564
6	508
7	455
8	293
9	215
10	160



- Draw a scatter plot of the **number of rentals** versus the week.
- Draw a line of best fit for the data.

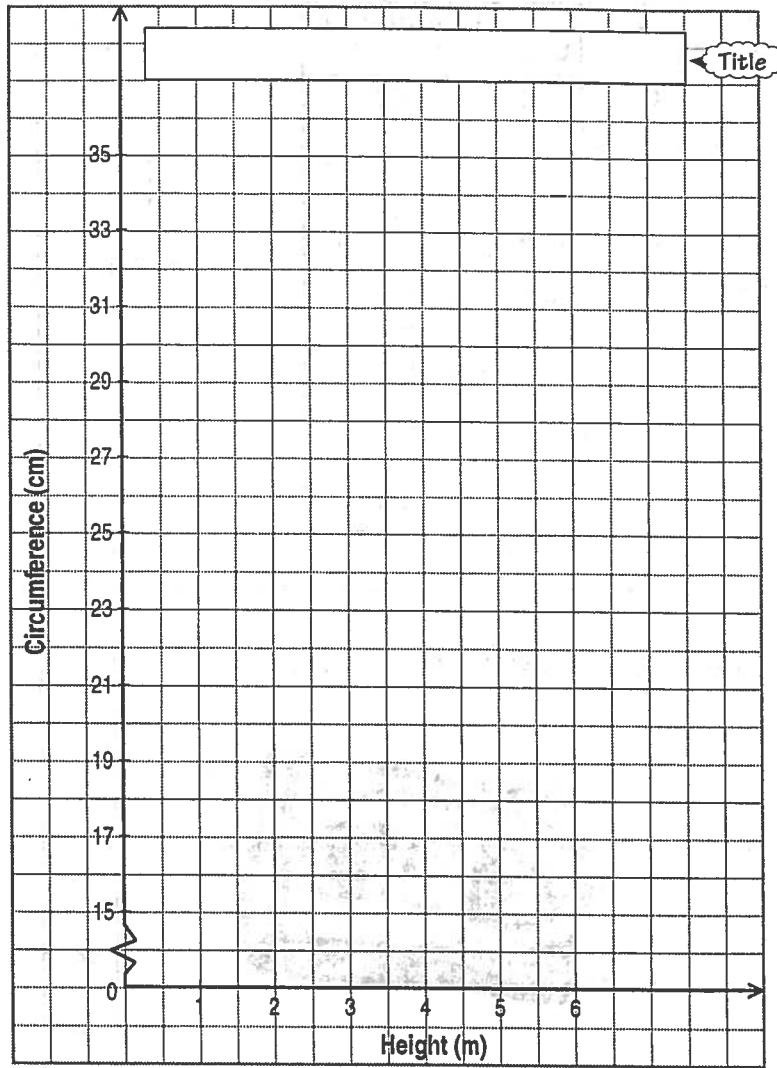
c) Describe the relationship.

d) Estimate the number of rentals in the 12th week.



3. The table shows the height and the circumference of a tree at different ages.

Age (years)	Height (m)	Circumference (cm)
1	1.1	15.2
2	1.1	18.2
3	2.4	20.7
4	2.5	23.2
5	3.1	27.0
6	4.3	29.5
7	4.5	32.0
8	5.3	33.9



- a) Plot the points on the graph.
- b) Draw a line of best fit.
- c) Describe the relationship.

d) Estimate the circumference for a height of 6 m.

e) Estimate the height for a circumference of 25 cm.

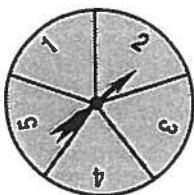
9.4 The Probability Formula

Probability Formula

$$P = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

Practice

1. What is the probability of spinning each of the following numbers with this spinner?



a) 1

$$P(1) = \frac{\square}{5}$$

b) 5

P(5)

c) 2

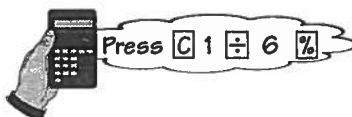
d) a prime number

e) 1, 2, 3 or 4

f) 6

2. What is the probability of rolling each of these numbers with a die? Express each answer as a percent.

Example:
 What is the probability of rolling a 4?
 There is one 4.
 So, $P(4) = \frac{1}{6}$
 $\approx 16.7\%$




a) 2

There is one 2.

$$P(2) = \frac{\square}{\square}$$

\approx _____ %

b) an odd number

There are _____ odd numbers.

$$P(\text{odd}) = \frac{\square}{\square}$$

\approx _____ %

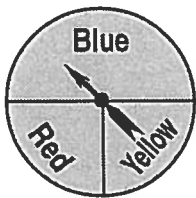
number of odd numbers
total number of sides

c) 4, 5, or 6

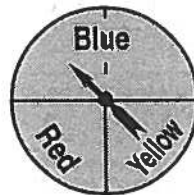
d) a number from 1 to 6

Problems and Applications

3. a) On this spinner, what is $P(\text{blue})$?



First: Make all parts equal in size.



There are parts of blue.

Second: So, $P(\text{blue}) = \frac{\square}{4}$ or **Reduce!**

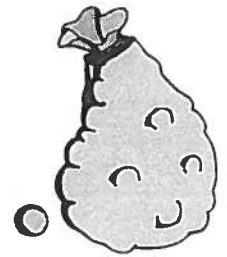
b) What is $P(\text{red})$?

c) How many times more likely is the spinner to land on blue than on red?

4. There are 4 green, 15 red, 6 yellow, and 5 black marbles in a bag. You remove 1 marble without looking.

a) What is the total number of possible outcomes?

Hint: Find the total number of marbles.



b) What is P(red)?

c) What is P(black)?

$$\begin{aligned}
 P(\text{red}) &= \frac{\text{number of red marbles}}{\text{total number of possible outcomes}} \\
 &= \frac{\boxed{}}{\boxed{}} \\
 &= \boxed{} \quad \leftarrow \text{Reduce!}
 \end{aligned}$$

d) What is P(purple)?

e) What is P(green, yellow, red, or black)?

5. **I M P O S S I B L E**

Each letter of the word **IMPOSSIBLE** is on a different card. All the cards are the same size. The cards are shuffled and placed face down on a table.

State the probability of drawing each of the following letters.

a) I

b) N

$$\begin{aligned}
 P(\text{I}) &= \frac{\text{number of Is}}{\text{total number of possible outcomes}} \\
 &= \frac{\boxed{}}{\boxed{}} \\
 &= \boxed{} \quad \leftarrow \text{Reduce!}
 \end{aligned}$$

How many letters in IMPOSSIBLE?

c) S

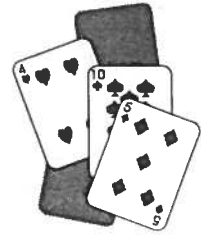
d) L

6. State the **probability** of drawing these cards from a standard deck of 52 play cards.

a) the 2 of clubs



b) a black card



$$P(\text{2 of clubs}) = \frac{\text{number of 2 of clubs}}{\text{total number of cards}}$$

$$= \frac{\boxed{}}{\boxed{}}$$

Reduce!

c) a heart

d) a red jack

← Reduce! →

7. Draw a spinner that gives the following probabilities.

$$P(\text{red}) = \frac{1}{8}$$

$$P(\text{blue}) = \frac{3}{8}$$

$$P(\text{green}) = \frac{1}{2}$$

Colour your spinner.

8. List two examples of probability in the media.



9.5 Independent Events

$$P(A \text{ and } B) = P(A) \times P(B)$$

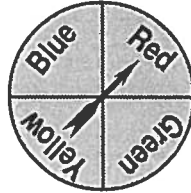
Problems and Applications

1. The spinner and the die are used for an experiment. Estimate each probability.

a) spinning red

$$P(\text{red}) = \frac{\square}{\square}$$

number of red parts
total number of parts



b) rolling a 5

$$P(5) = \frac{\square}{\square}$$



c) spinning red and rolling five

$$P(\text{red and } 5) = P(\text{red}) \times P(5)$$

$$= \frac{\square}{\square} \times \frac{\square}{\square}$$

Substitute

$$= \frac{\square}{\square} \times \frac{\square}{\square}$$

Multiply

$$= \underline{\hspace{2cm}}$$

Hint:
Use your
answers from
a) and b).

d) rolling an odd number and spinning green

First: $P(\text{odd}) = \frac{\square}{\square}$

number of odds
total number of sides

$$P(\text{green}) = \frac{\square}{\square}$$

$$= \frac{\square}{\square}$$

Reduce!

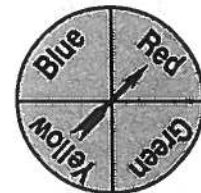
Second: $P(\text{odd and green}) = P(\text{odd}) \times P(\text{green})$

$$=$$

Substitute

Multiply

e) rolling an even number and not spinning a blue



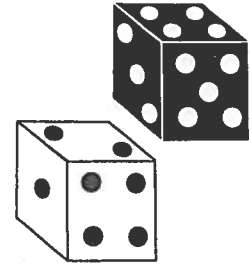
2. A white die and a black die are rolled. What is the probability of each outcome?

a) 6 on the white die and 1 on the black die

$$P(\text{white 6 and black 1}) = P(6) \times P(1)$$

$$= \frac{\square}{\square} \times \frac{\square}{\square}$$

$$= \underline{\hspace{2cm}}$$



b) 6 on each die

c) an even number on the white die and an odd number on the black die

There are even numbers on the white die.

There are odd numbers on the black die.

$$P(\text{even}) = \underline{\hspace{2cm}}$$

$$P(\text{odd}) = \underline{\hspace{2cm}}$$

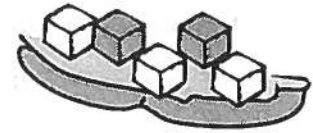
$$\text{So, } P(\text{even and odd}) = \underline{\hspace{4cm}}$$

=

=

d) an even number on the black die and an even number on the white die

3. A bag contains 3 white cubes and 2 grey cubes. Each cube is replaced after it is drawn.
What is each probability?



a) a white cube, then a grey cube

$$P(\text{white}) = \frac{\boxed{}}{\boxed{}}$$

number of white cubes
total number of cubes

$$P(\text{grey}) = \frac{\boxed{}}{\boxed{}}$$

number of grey cubes
total number of cubes

$$P(\text{white and grey}) = P(\text{white}) \times P(\text{grey})$$

$$= \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

b) 2 white cubes

$$P(\text{white}) = \frac{\boxed{}}{\boxed{}}$$

Remember
2
events.

$$P(\text{white and white}) = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

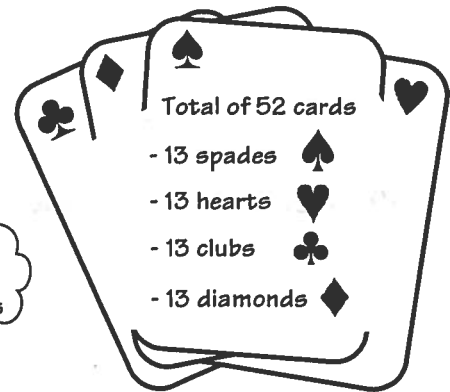
4. What is the probability of choosing a club (♣) from each set of cards?

a) the full deck

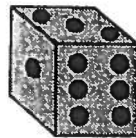
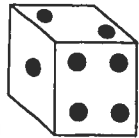
b) all the black cards

c) all the red cards

Hint:
of clubs
of black cards



5. A black die, a grey die, and a white die are rolled.



Calculate each probability.

a) a 5 on all 3 dice

$$P(5 \text{ on white die}) = \frac{\quad}{6}$$

$$P(5 \text{ on grey die}) = \quad$$

$$P(5 \text{ on black die}) = \quad$$

$$P(5 \text{ on white die and } 5 \text{ on grey die and } 5 \text{ on black die}) = P(5 \text{ on white die}) \times P(5 \text{ on grey die}) \times P(5 \text{ on black die})$$

$$= \quad \times \quad \times \quad$$

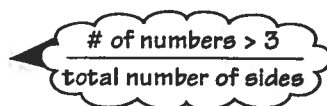
$$= \quad$$



b) a 2 on the white die, a 3 on the grey die, and a 6 on the black die

c) a number greater than 3 on the black die, an even number on the grey die, and an odd number on the white die

$$P(> 3) =$$



$$P(\text{even}) =$$

$$P(\text{odd}) =$$

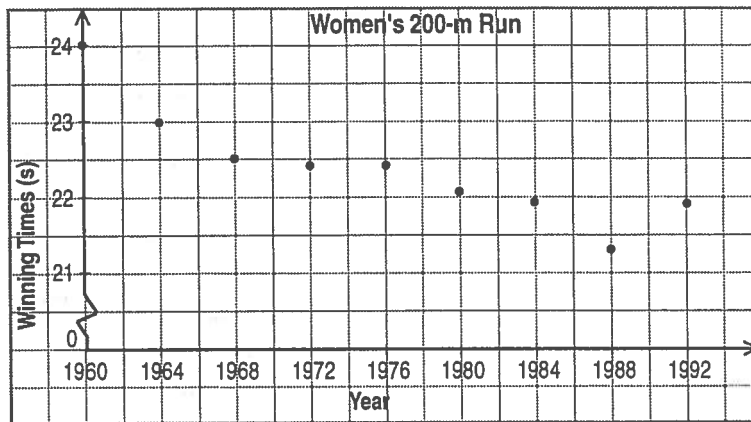
$$P(> 3 \text{ and even and odd}) =$$



Review



1. The scatter plot shows the winning times in the women's 200-m run at the Olympic Games in different years.



- a) What was the approximate winning time in 1976? _____
- b) In what year was the winning time closest to 22.5 s? _____
- c) About how many seconds longer did the race take in 1960 than it did in 1988?

Winning Time in 1960 → _____

Winning Time in 1988 → _____

Subtract!

Difference → _____

Sentence: _____

- d) What has happened to the winning times from 1960 to 1988?

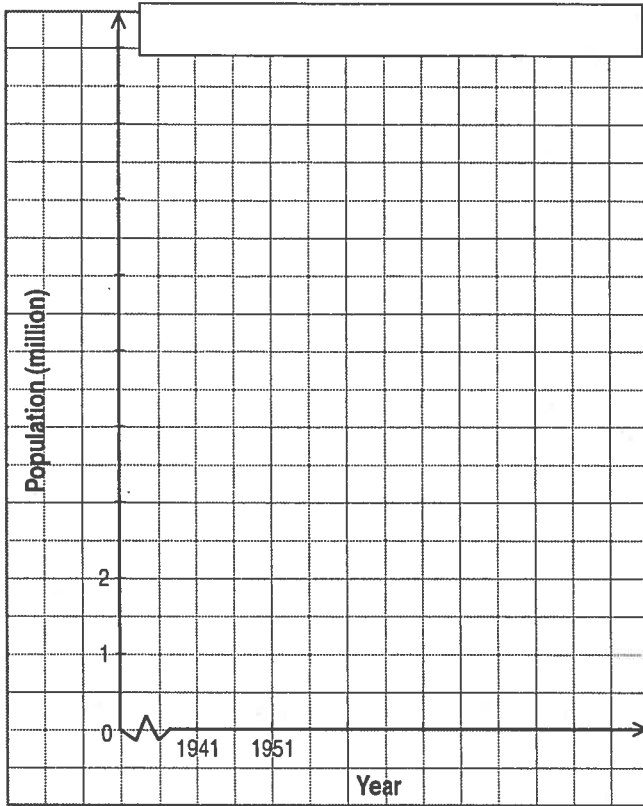
- e) Use your research skills to find the winning times in the women's 200-m run in the 2000 Olympic Games.



2. The table shows the total population of Canada's four Western Provinces in different years.

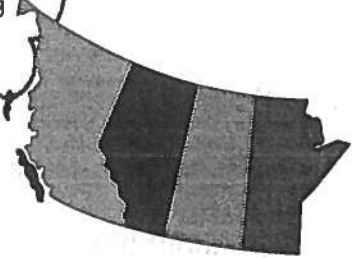
Year	Population (nearest 0.1 million)
1941	3.2
1951	3.7
1961	4.8
1971	5.7
1981	7.0
1991	7.9

a) Display the data on a scatter plot.



Remember to:

1. Give the graph a title.
2. Finish labeling each axis.
3. Plot the population.



b) Draw a line of best fit.

Line of Best Fit

- a **straight line** on a graph that lies as close as possible to all the points on the graph
- There are about as many **points above** the line as there are **below** the line.

c) Describe the relationship between the population and the year.

d) Estimate the total population of the four Western Provinces

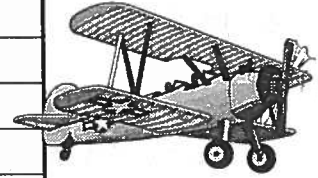
i) in 1966. _____

ii) in 2001. _____

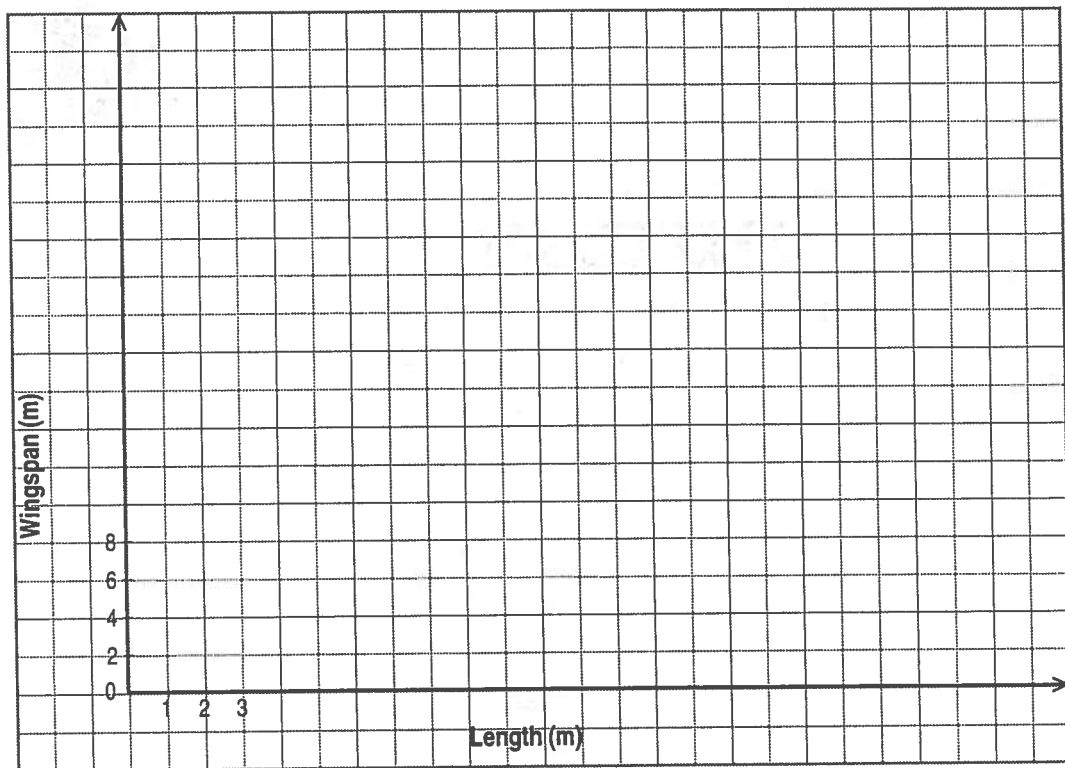
Extend line of best fit.

3. The table shows the lengths and wingspans of airplanes.

Plane (Year built)	Length (m)	Wingspan (m)
<i>Flyer</i> (1903)	6.43	12.29
<i>June Bug</i> (1908)	9.1	13.89
<i>Demoiselle</i> (1909)	6.1	5.5
<i>Blériot XI</i> (1909)	8	7.8
<i>Deperdussin Racer</i> (1912)	6.1	6.65
<i>Grand</i> (1913)	20.02	28.02
<i>Junkers J-1</i> (1915)	9.04	16.8
<i>Fokker DVII</i> (1918)	7.01	8.94
<i>Ford Trimotor</i> (1926)	15.19	22.6
<i>Lockheed Vega</i> (1927)	8.38	12.5



a) Display the data on a scatter plot.



b) Draw a line of best fit.

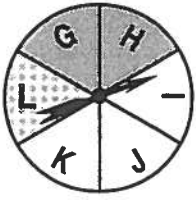
c) Estimate the wingspan of an airplane with a length of 10 m.

Interpolate

d) Estimate the length of an airplane with a wingspan of 30 m.

Extrapolate
→ extend the line
to estimate.

4. Find each probability.



a) $P(H) = \frac{\square}{\square}$

number of Hs
total number of outcomes

number of sections
on spinner

b) $P(H \text{ or } I)$

c) $P(\text{grey})$

d) $P(\text{black})$

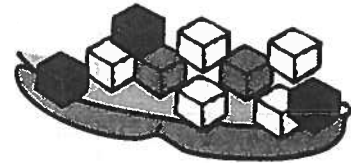
e) $P(\text{grey or white})$

f) $P(G, H, \text{ or } I)$

g) $P(G, H, I, J, K, \text{ or } L)$

5. A bag contains 3 black cubes, 5 white cubes, and 2 grey cubes. What is the probability of drawing each of the following if each cube is replaced before the next draw?

a) a white cube



b) a black cube, then another black cube

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$P(\text{Black and Black}) = P(\text{Black}) \times P(\text{Black})$$

$$= \frac{\text{number of black cubes}}{\text{total number of cubes}} \times \frac{\text{number of black cubes}}{\text{total number of cubes}}$$

$$= \frac{\square}{10} \times \frac{\square}{\square}$$

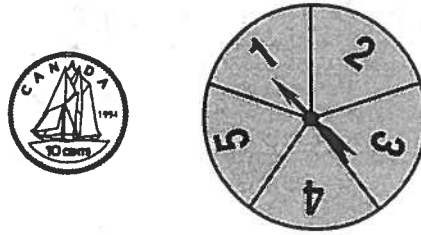
$$= \frac{\square}{\square}$$

c) a yellow cube

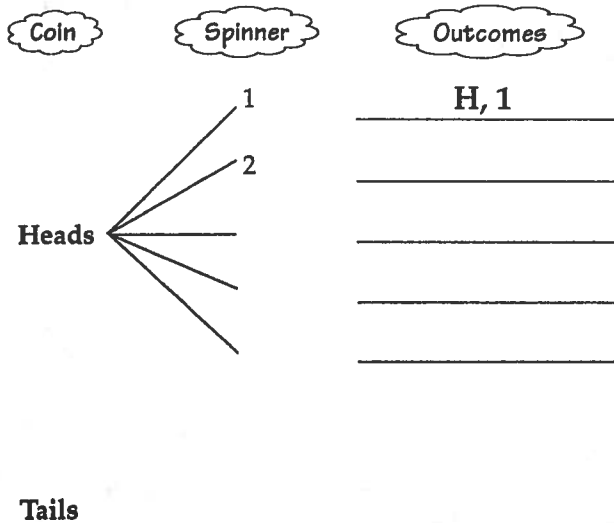
d) a white (W), then a black (B), then a grey (G) cube

$$P(W \text{ and } B \text{ and } G) = P(W) \times P(B) \times P(G)$$

6. This spinner is used for an experiment in which you toss a dime and spin the spinner.



a) How many possible outcomes are there?



Sentence: _____

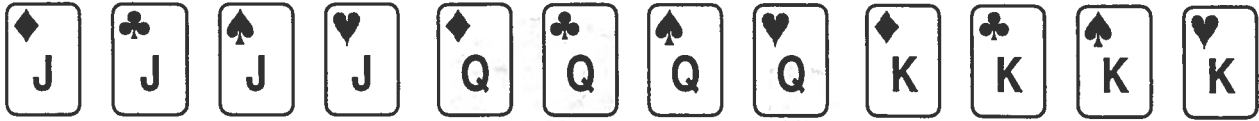
b) What is the probability of tossing a head and spinning a 4?

$$P = \frac{\text{number of (H, 4)}}{\text{total number of outcomes}}$$

c) What is the probability of tossing a tail and spinning an odd number?

$$P = \frac{\text{number of possible outcomes}}{\text{total number of outcomes}}$$

7. Three cards are drawn from the 12 cards below.



What is the probability of drawing the following if each card is replaced before the next draw?

a) 3 kings

First: Find $P(K)$.

$$P(K) = \frac{\boxed{}}{12}$$

← *number of Kings*
total number of cards

$$= \boxed{}$$

Reduce!

Second: Find $P(3 \text{ Kings})$

$$P(3 \text{ Kings}) = P(K) \times P(K) \times P(K)$$

$$= \boxed{} \times \boxed{} \times \boxed{}$$

$$=$$

b) 3 clubs (♣)

First: Find $P(1 \text{ club } \clubsuit)$.

$$P(\clubsuit) = \frac{\boxed{}}{\boxed{}}$$

number of ♣
total number of cards

Reduce!

Second: Find $P(3 \text{ clubs } \clubsuit)$.

$$P(3 \clubsuit) = P(\clubsuit) \times P(\clubsuit) \times P(\clubsuit)$$



c) a queen ♦, then a jack ♦, then a 6 ♦.

First: Find $P(Q \spadesuit)$, $P(J \spadesuit)$, and $P(6 \spadesuit)$.

$$P(Q \spadesuit) = \boxed{}, P(J \spadesuit) = \boxed{}, \text{ and } P(6 \spadesuit) = \boxed{}.$$

Second: $P(Q \spadesuit) \text{ and } P(J \spadesuit) \text{ and } P(6 \spadesuit) = P(Q \spadesuit) \times P(J \spadesuit) \times P(6 \spadesuit)$



$$= \underline{} \times \underline{} \times \underline{}$$

$$= \underline{}$$

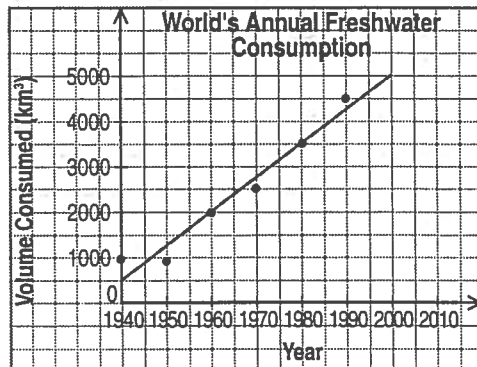
d) a queen, then a jack, then a king

Do this question on looseleaf.

Chapter Check



1. The scatter plot shows how the world's annual freshwater consumption changed over 50 years.



a) About how much water was used in

i) 1940? _____

ii) 1965? _____

iii) 1980? _____

iv) 1995? _____

b) During which year was about 3000 km³ of water used? _____

c) About how much more water was used in 1990 than in 1940?

Subtract

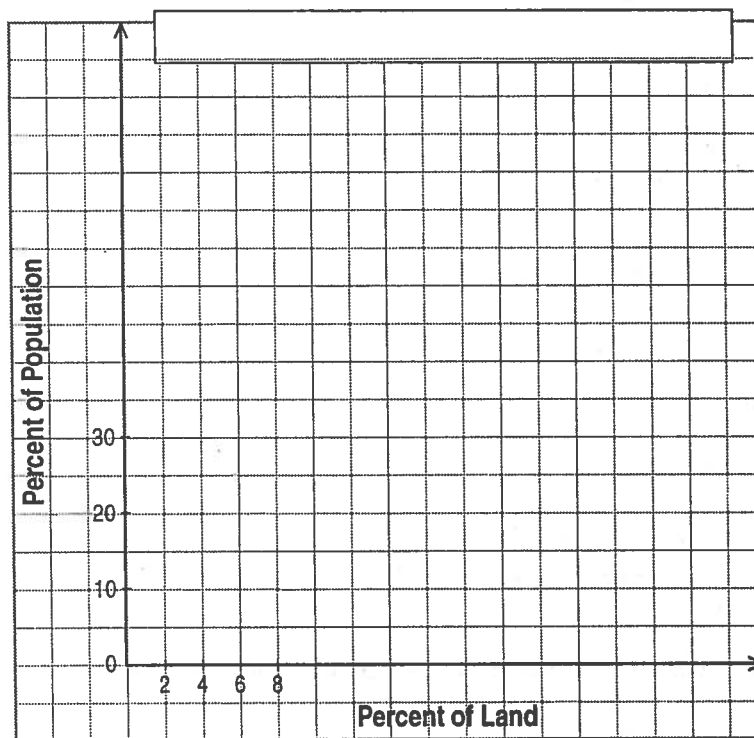
d) Estimate how much water might be used in 2015.

Extend the line of best fit.

2. The table shows the percent of the world's land and the percent of the world's population on each continent.

Continent	Percent of Land	Percent of Population
Africa	20	12
Antarctica	9	0
Asia	30	60
Australia	5	0.3
Europe	7	14
N. America	16	8
S. America	12	5

a) Display the data on a scatter plot.



b) Is there a relationship shown in the scatter plot. Explain.

3. Find the number of possible outcomes for each of the following.

a) when you roll a die _____

b) when you toss a coin _____

c) when you roll a die and toss a coin



Coin

Die

Outcomes

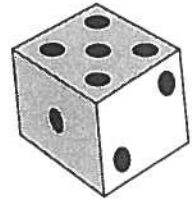
Heads

1

2

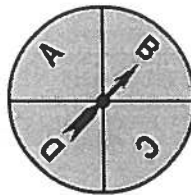
3

H, 1

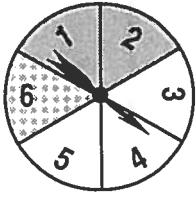


Tails

d) when you toss a coin and spin the spinner



4. Find each probability.



a) $P(1) = \frac{\square}{\square}$

number of 1s
total number of outcomes

b) $P(\text{white})$

c) $P(1 \text{ or } 2)$

d) $P(\text{red})$

Reduce!

e) $P(5 \text{ or } 6)$

f) $P(\text{grey, white, or dotted})$

g) $P(1, 2, 3, 4, 5, \text{ or } 6)$

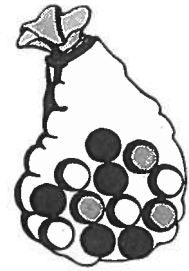
5. A bag contains 4 white marbles, 5 black marbles, and 3 grey marbles. Each marble is replaced before the next draw. What is the probability of each of the following?

a) a white marble

b) a grey marble

$P(\text{white}) = \frac{\square}{\square}$

number of white marbles
total number of marbles



Reduce!

= _____

c) a white marble, then a grey marble

$P(\text{white and grey}) = P(\text{white}) \times P(\text{grey})$

d) 3 grey marbles

$P(3 \text{ grey}) = P(\text{grey}) \times P(\text{grey}) \times P(\text{grey})$

